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Producing a Scale to Indicate the Volume of Liquid in a Container

(Menghasilkan Satu Skala untuk Menunjukkan Isipadu Cecair dalam Satu Bekas)

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ABSTRACT

The present work considers the volume-depth relationship for several container shapes such as cylinders, spheres and cones. The evaluation of the volume corresponding to a given depth is easily carried out on a spreadsheet. A computer program is described for carrying out the inverse process, that is, to output values of depth corresponding to selected volumes, hence producing a depth-gauge. The calculation can take into account the shape of the container and how the divisions (numbered and un-numbered) should be distributed on the scale. If the scale is graduated as a fraction of the total volume and displayed at an arbitrary size on the monitor screen, then "universal" results are obtained, for example for any sphere, or any cylinder, etc., independent of its actual dimensions. In addition, if the volume is specified in litres (or gallons, or any other units) and the length of the depth-scale is specified, then a gauge can be produced that will suit that particular container. It can be displayed on the screen only as a scale drawing, but exact dimensions can be output as a file, which will allow the full-size gauge to be drawn, either using drawing-office equipment or an automatic drawing machine.

Keywords: Cones; cylinders; spheres; volume-depth relationship

ABSTRAK

Kerja ini mempertimbangkan hubungan antara isipadu dan kedalaman untuk beberapa bentuk bekas seperti selinder, sfera dan kon. Penilaian isipadu untuk satu kedalaman yang diberikan, adalah mudah dilakukan pada satu helaian hamparan. Satu program komputer dihuraikan untuk melakukan proses songsangan, iaitu, memberikan nilai kedalaman untuk isipadu yang dipilih, oleh yang demikian menghasilkan satu tolok kedalaman. Pengiraan boleh mengambil kira bentuk bekas itu dan bagaimana bahagian (bernombor atau tidak bernombor) perlu diagihkan pada skala. Jika skala adalah bergraduat sebagai satu pecahan isipadu total dan dipamerkankan pada satu saiz sembarangan pada layar monitor, maka keputusan "semesta" diperolehi, sebagai contoh untuk sebarang sfera, atau sebarang selinder, dan lain-lain, bebas dari dimensi sebenarnya. Sebagai tambahan, jika isipadu dinyatakan dalam liter (atau gelen, atau unit lain) dan panjang skala kedalaman dinyatakan, maka satu tolok boleh dihasilkan untuk disesuaikan dengan bekas tertentu. Skala ini boleh dipamerkan pada skrin hanya sebagai satu skala lukisan, tetapi dimensi yang tepat boleh dikeluarkan sebagai satu fail, yang akan membenarkan tolok saiz sebenar dilukis, sama ada menggunakan peralatan lukisan pejabat atau mesin pelukis otomatik.

Kata kunci: Hubungan isipadu dan kedalaman; kon; selinder; sfera

Introduction

Many container shapes bear a label showing their total volume when they are full. This total volume may be determined in general by three methods (1) by displacement: this is for any shape. For example, a typical kettle, like one we may own at home, can be filled with water. The amount of water that this kettle contains gives the total volume. So an experimental approach could be used to verify the total volume, (2) by algebraic formula: this can be used for simple container shapes. For example, shapes like vertical cylinder, sphere, hemisphere, cone, horizontal cylinder, horizontal cylinder with hemi-spherical ends, and truncated cone. The total volume formulas for these eight shapes are given in Table 1, (3) by integral calculus: this is for any shape provided that a formula exists for its boundary.

It is important to note here that the container's orientation has no effect on this total volume. For example, for a cylinder, the total volume is the same whether it is horizontally or vertically mounted.

Frequently, when the container is partially filled, it is important to know the level of the volume of liquid. The information of this partial volume is normally indicated by a volume gauge. Several methods are normally used to produce a scale to indicate the volume of liquid in a container.

1. By displacement: for example, a modern plastic electric kettle often has a volume gauge on a vertical transparent tube attached to it, giving the volume in terms of the number of cups of tea. If the kettle does not have the form of a vertical cylinder, then the graduations on the volume gauge are not uniformly

TABLE 1. The eight containers with their configurations considered in this work. The well-known total volume formula will be used in Table 2

Container symbol	Shape	Dimensions			Total volume formula V
		Total depth	Radius	Extra	
C1	Vertical cylinder	b	a	–	$\pi a^2 b$
C2	Sphere	$2a$	a	–	${}^4/_3\pi a^3$
C3	Hemisphere	a	a for bottom plane	–	${}^2/_3\pi a^3$
C4	cone (point up)	b	a for bottom plane	–	${}^1/_3\pi a^2 b$
C5	cone (point down)	b	a for top plane	–	${}^1/_3\pi a^2 b$
C6	horizontal cylinder	$2a$	a	–	$\pi a^2 L$
C7	horizontal cylinder with hemi-spherical ends	$2a$ for central cylinder	a for central cylinder	$m = a/L$, where L = length of central cylinder, a = depth of end-caps	$\pi a^2(L + {}^4/_3 a)$
C8	truncated cone	b	a for bottom plane	$k = a'/a$, where a' = radius of top plane	$\pi b(a^2 + aa' + a'^2)/3$

spaced. Somebody must therefore have solved the problem of relating volume to depth – possibly using an "experimental" approach, in which they added water, one cupful at a time, and put marks on the depth gauge, and then used this as a "master" from which to develop a mass-produced version. This same experimental technique with the use of vertical transparent tube may also be applied to the simple external rainwater tank gauge (home.iprimus.com 2008), where the top of the tube would be open to the air. In cases where the container is pressurised, or its contents are volatile or toxic, the top of the tube would be connected to the top of the container.

2. By electronic gadgets: various methods of electronic (remote) read-out are also possible. A pressure transducer at the bottom of the container would give a signal proportional to the depth of liquid and, in the case of a pressurised container, two transducers, at the top and bottom, would be needed to give the pressure difference(O'Shea 2004). The most popular method is a float connected to an electrical potentiometer. This combines the non-linearity of the volume-depth relationship with an additional non-linearity because the float follows a circular arc rather than a vertical line (Neeser & Kuechenmeister 2002).

In contrast to the total volume, the volume gauge to indicate the partial volume is affected by the container's orientation. For example, the gauge has a different scale for a horizontally-mounted cylinder, than for a vertically mounted one.

There are many web-sites which enable visitors to calculate on-line the total volume and partial volumes (Neeser & Kuechenmeister 2002; Mathguide 2008; To 2008; Lutus 2009; AquaDyn 2008). There are web-sites informing visitors of their patented volume-gauge devices (O'Shea 2004; Neeser & Kuechenmeister 2002). These volume gauges are not necessarily meant for liquids; devices for gases (home.iprimus.com 2008; O'Shea, 2004; Shouman 2004) and liquefied gases (Bryukhanov & Grigorovskii 1976) are also reported. All these web-sites or reports however do not produce a scale to indicate the volume of liquid in a container. We discuss here a unified procedure to produce this scale for the eight container shapes given in Table 1. This is done by theoretical deduction of the volume-depth relationship.

MATERIALS AND METHOD

Eight container shapes are considered in this work. Their names and dimensions are given in Table 1. The dimensions will then be used:

1. to state the well-known total volume formula V. This is shown in the last column of Table 1,
2. to derive the relationship between fractional total volume $F(x)$ and the fractional total depth $f(x)$, where x is the depth of liquid, measured from the container's lowest point. This is shown in the last column of Table 2.

We shall now describe the method used to get $F(x)$ in terms of $f(x)$ using the following steps:

1. we get $f(x)$, the fraction of total depth f at depth x,
2. we get $r(x)$, the horizontal plane radius r at depth x, for those cases where the cross-section is circular
3. we get $A(x)$, the horizontal plane area A at depth x, except where it is easier to use a geometrical method to get the volume directly
4. we get $V(x)$, the volume formula at depth x, and
5. lastly we divide $V(x)$ by V, then write the results in terms of f, to get F.

Steps 1, 2 and 3 are usually straightforward geometry. Step 5 on the other hand is simple mathematics. To perform step 4, however we use integral calculus. To explain this integration, we consider a container of arbitrary shape, filled to a depth x, as shown in Figure 1. If we now add a small additional volume δV, to $V(x)$, the depth increases by δx, and we see that $\delta V = A(x)\delta x$. As $\delta x \to 0$, we get the derivative:

$$\frac{dV}{dx} = A(x).$$

Therefore

$$V(x) = \int_0^x A(x')dx'.$$

Note that the variable x' is introduced here to distinguish the limit x from the quantity that runs from 0 to x. Although we can describe this process in words as "Integrate the area with respect to x, and substitute in the limits 0 and x...", we should be aware that this description uses one symbol, x, with two different meanings.

VERTICAL CYLINDER C1, SPHERE C2, HEMISPHERE C3, CONE (POINT UP) C4 AND CONE (POINT DOWN) C5

Using the method described above, the five steps listed in section previous for these five shapes are shown in Table 2.

TO GET f IN TERMS OF f, FROM ALL THE FIVE STEPS

We take the sphere C2 as an example here. For this shape, $f(x)=x/2a$. The cross-section at a depth x will be a circle with radius $r = \sqrt{[a^2 - (a-x)^2]}$ and area $A(x) = \pi[a^2 - (a-x)^2] = \pi(2ax - x^2)$. Integrating with respect to x, and substitutiog in the limits 0 and x, we get $V(x) = \pi(ax^2 - x^3/3)$. The expression for the fractional volume is $F(x) = V(x)/V(2a) = \pi(ax^2 - x^3/3)/(^4/_3\pi a^3) = ¾[4(x/2a)^2 - 8(x/2a)^3/3] = 3f^2 - 2f^3$.

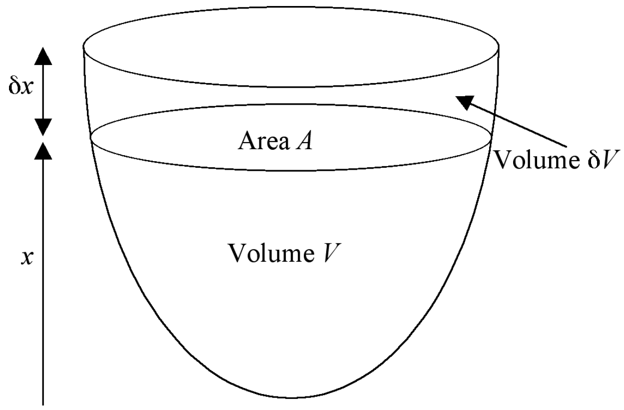

FIGURE 1. Illustrating the relation between cross-sectional area and volume

To verify V from $V(x)$ we take the same C2. Table 1 gives the total volume of a sphere as $^4/_3\pi a^3$. Column 5 in Table 2 gives $V(x) = \pi(ax^2 - x^3/3)$ for a sphere. Substituting $x = 2a$ as the full depth into this $V(x)$, we therefore verify that the total volume $V(2a) = \pi[a(2a)^2 - (2a)^3/3] = \pi a^3(4 - {^8/_3}) = {^4/_3}\pi a^3$.

CONTAINER C3 AND SOME GENERALISATIONS

Hemisphere C3 is a special case of a semi-ellipsoid. We purposely do not include this semi-ellipsoid in the table, as this shape has the same $F(f)$ as the hemisphere C3. In other words, this is a universal relation, applying to semi-ellipsoids of any proportions, including a

TABLE 2. The steps taken in order (from left column to right column), to derive the dependence of fraction of total volume F on the fraction of total depth f, for each shape

Container symbol	The horizontal plane at depth x			Volume formula at depth x, $V(x)$	F[a]
	$f(x)$	$r(x)$	$A(x)$		
C1	x/b	a	πa^2	$\pi a^2 x$	f
C2	$x/2a$	$\sqrt{[a^2 - (a-x)^2]}$	$\pi(2ax - x^2)$	$\pi(ax^2 - x^3/3)$	$3f^2 - 2f^3$
C3	x/a	$\sqrt{(a^2 - x^2)}$	$\pi(a^2 - x^2)$	$\pi(a^2x - x^3/3)$	$(3f - f^3)/2$
C4	x/b	$a(1 - x/b)$	$\pi(a/b)^2(b^2 - 2bx + x^2)$	$\pi(a/b)^2(b^2x - bx^2 + x^3/3)$	$1 - (1-f)^3$[b]
C5	x/b	ax/b	$\pi(ax/b)^2$	$\pi(a/b)^2x^3/3$	f^3[b]
C6	$x/2a$, or $(1-\cos\theta)/2$	–[c]	–[c]	$½a^2L\times(2\theta - \sin 2\theta)$	$(2\theta - \sin2\theta)/(2\pi)$
C7	$x/2a$, or $(1-\cos\theta)/2$	$\sqrt{[a^2 - (a-x)^2]}$[d]	$\pi(2ax - x^2)$[d]	$½a^2L\times(2\theta - \sin2\theta) + \pi(ax^2 - x^3/3)$	$\{^1/_{2\pi}(2\theta - \sin2\theta) + m[(1-\cos\theta)^2 - (1-\cos\theta)^3/3]\}/(1 + {^4/_3}m)$
C8	x/b	$a'x/b + a(1 - x/b)$	$\pi[b + (a' - a)x/b]^2$	$\pi/b^2[b^2a^2x + ba(a'-a)x^2 + (a'-a)^2x^3/3]$	$[3f + 3(k-1)f^2 + (k-1)^2f^3]/(1 + k + k^2)$

[a]This is equal to $V(x)/V$, in which V is given in Table 1.
[b]Both of these relationships also apply to cones with elliptical cross-sections and pyramids.
[c]This is irrelevant as the volume is more easily calculated directly.
[d]This is only the second part of the shape namely the two hemispherical endcaps. The first part (central cylinder) is irrelevant here.

hemisphere. An ellipsoid can be regarded as a sphere that has been stretched in one or more dimensions, and this transformation does not change the fractional volume F or the fractional depth f, so it should not be surprising that the result is the same. Alternatively, it is possible to treat the semi-ellipsoid from first principles. If we consider a semi-ellipsoid with total depth b, and radius (of the bottom plane) a, (i.e. the three semi-axes are a, a and b), then the radius r and depth x are related by the standard equation for an ellipse: $r^2/a^2 + x^2/b^2 = 1$. The total volume is ${}^2/_3\pi a^2 b$. For this shape, $f = x/b$, $r = (a/b)\sqrt{(b^2 - x^2)}$, $A(x) = \pi(a/b)^2(b^2 - x^2)$, $V(x) = \pi(a/b)^2(b^2x - x^3/3)$. The expression for the fractional volume is $F(x) = V(x)/V(b) = \pi(a/b)^2(b^2x - x^3/3)/({}^2/_3\pi a^2 b) = {}^3/_2[x/b - {}^1/_3\,(x/b)^3] = (3f - f^3)/2$. This $F(x)$ is the same as we obtained for the hemisphere. Notice also that the total volume ${}^2/_3\pi a^2 b$ may be verified if we substitute $x = b$, the total depth, into $V(x) = \pi(a/b)^2[b^2x - x^3/3]$.

HORIZONTAL CYLINDERS C6 AND C7

These are respectively the horizontal cylinder with plane ends and horizontal cylinder with spherical end-caps. These two shapes are dealt with separately because the horizontal plane through these two shapes at depth x cannot be described in terms of the radius. This is the reason why the results of steps 2 and 3 for these shapes in Table 2 are left empty.

To get $V(x)$ for these two shapes, we can get the horizontal area $A(x)$ at depth x, then integrate with respect to x:

1. For C6: the horizontal area is $A(x)= 2L\sqrt{[a^2 - (a - x)^2]}$. This case is more difficult to integrate, because the square root remains in the integral, but the substitution $\cos\theta = (a - x)/a$ allows the integration to be completed.
2. For C7: the horizontal area is $A(x)= 2L\sqrt{[a^2 - (a - x)^2]} + \pi[a^2 - (a - x)^2]$, where the first part is the surface area contained in the central cylinder and second part is the area contained in the two hemispheres of radius a added to the ends. As above, the first part is difficult to integrate although the second part is identical to the case of the sphere, C2.

The above problem of integration can be avoided by using a geometrical approach. In Figure 2, representing the cross-section of the container, OA = OB = OC = OE = a, DE = x, and AB is the surface of the liquid.

We want the area of ADBE. This is equal to (area of the sector OAEB) – (area of the triangle OADB). Letting θ represent the angle AOE (= BOE), we use the well-known formulas for the area of a sector and the area of a triangle, to get the result $2 \times ½\,\theta\, a^2 - ½\, a^2 \sin 2\theta = ½\, a^2(2\theta - \sin 2\theta)$. $\cos\theta = (a - x)/a$. Bearing in mind for this shape $f = x/2a$, we can write $\cos\theta = 1 - x/a = 1 - 2f$.

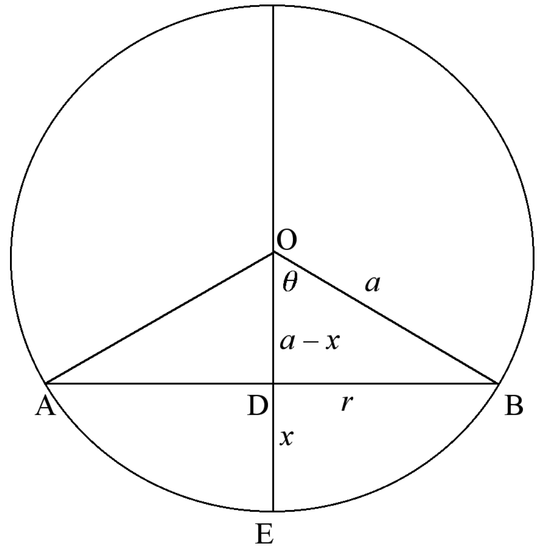

FIGURE 2. Cross-section of horizontal cylinder, showing liquid level at AB

HORIZONTAL CYLINDER WITH PLANE ENDS C6

The volume at depth x is therefore $V(x)$ = (length L)×(area of segment ADBE) = $½\, a^2L(2\theta - \sin 2\theta)$. We then obtain $F(x) = V(x)/V(b) = (2\theta - \sin 2\theta)/(2\pi)$. Although we can now express this in terms of x, this does not help with the numerical evaluation, because in a practical calculation we would first find the value of θ. Similarly the relationship between the dimensionless quantities F and f can be obtained by substituting $f = (1 - \cos\theta)/2$, but again a two-step evaluation process is simpler to carry out.

HORIZONTAL CYLINDER WITH SPHERICAL END-CAPS C7

This is a common choice for a container of pressurised liquids or gases (although it is only for the case of a liquid that a depth gauge would be relevant). To find the volume at depth x, we add the previous result $V(x) = ½\, a^2L(2\theta - \sin 2\theta)$ to the result for the sphere $V(x) = \pi[ax^2 - x^3/3]$, to obtain $V(x) = ½\, a^2L(2\theta - \sin 2\theta) + \pi[ax^2 - x^3/3]$.

In this case, the dependence of F on f will not be a universal relationship, as the problem contains a dimensionless parameter $m = a/L$, giving a family of graphs of F against f. Given a particular value of m, we can imagine the container compressed (or stretched) along its axis, so that the end-caps become semi-ellipsoids. The parameter m therefore has a wider interpretation: the ratio of the depth of the ellipsoidal end-caps to the length of the cylinder. To see the dependence on m more clearly, we substitute $x = a(1 - \cos\theta)$ and $a = mL$ in the expression for $V(x)$, and divide by the total volume $V(2a)=\pi a^2(L + {}^4/_3 a)$ to get

$$F = \{{}^1/_{2\pi}\, L(2\theta - \sin 2\theta) + mL[(1 - \cos\theta)^2 - (1 - \cos\theta)^3/3]\}/(L + {}^4/_3 mL)$$
$$= \{{}^1/_{2\pi}\,(2\theta - \sin 2\theta) + m[(1 - \cos\theta)^2 - (1 - \cos\theta)^3/3]\}/(1 + {}^4/_3 m),$$

where $f = ½(1 - \cos\theta)$.

We also note that pressure vessels often have concave end caps (like the base of an aerosol can). These cases can be included by giving m a negative value ($m > -0.5$, otherwise the two end caps will intersect).

TO VERIFY *V* FROM *V(x)* FOR C6 AND C7

Table 1 yields the total volume of containers C6 and C7 respectively as $\pi a^2 L$ and $\pi a^2(L + {}^4/_3 a)$. For C7, substituting x as the full depth $2a$, we will first get $\cos\theta = -1$, which means $\theta = \pi$. Using these values of $x = 2a$ and $\theta = \pi$ in $V(x) = ½\, a^2L(2\theta - \sin 2\theta)$, we then verify the total volume $V(2a) = ½\, a^2L(2\pi - \sin 2\pi) = \pi a^2L$. For C7, similarly substituting $x = 2a$ and $\theta = \pi$ in $V(x) = ½\, a^2L(2\theta - \sin 2\theta) + \pi[ax^2 - x^3/3]$, we get $V(2a) = ½\, a^2L(2\pi - \sin 2\pi) + \pi[a(2a)^2 - (2a)^3/3] = \pi a^2(L + {}^4/_3 a)$.

TRUNCATED CONE C8

The truncated cone, or frustum, has total depth b, with radii a and a' at the bottom and top respectively. The total volume $V = \pi b(a^2 + aa' + a'^2)/3$. The ratio $k = a'/a$ will appear as a parameter in this case.

TO GET *F* IN TERMS OF *f*, FOR C8, FROM ALL THE FIVE STEPS

The radius at depth x is $r(x) = a'x/b + a(1 - x/b)$, and the area at depths x is $A(x) = \pi[b + (a' - a)x/b]^2$. The volume is obtained by integrating $A(x)$, giving $V(x) = \pi/b^2[b^2a^2x + ba(a' - a)x^2 + (a' - a)^2x^3/3]$. Bearing in mind for this shape $x = bf$, $a' = ka$, the expression for the fractional volume is $F = V(x)/V(b)$:

$$F = \frac{3b^3a^2 f + 3b^3a^2 f^2 \left(k-1\right)^2 f^3}{b^3a^2\left(1+k+k\right)^2}$$

$$= \frac{3f + 3\left(k-1\right)f^2 + (k-1)^2 f^3}{1+k+k^2}.$$

Again, F is seen to be a function of f, with k appearing as a dimensionless parameter. For the particular cases $k = 0$, $k = 1$ and $k \to \infty$, this reduces to the results obtained above for a cone (point up), a vertical cylinder, and a cone (point down).

TO VERIFY *V* FROM *V(x)* FOR C8

Substituting $x = b$ in $V(x)$, we obtain

$$\begin{aligned} V(b) &= \pi/b^2[b^2a^2x + ba(a' - a)x^2 + (a' - a)^2x^3/3] \\ &= \pi/b^2[b^2a^2b + ba(a' - a)b^2 + (a' - a)^2b^3/3] \\ &= \pi b(a^2 + aa' + a'^2)/3. \end{aligned}$$

RESULTS

Figure 3 shows the variation of F with f for the various cases considered above, except for the horizontal cylinder with spherical ends and the frustum of a cone. The vertical cylinder is of course represented by a straight line. Of the others, the horizontal cylinder shows the least deviation from the straight line, followed by the sphere and the hemisphere and finally the two orientations of a cone. The results for the horizontal cylinder with spherical ends are shown in figure 4, for a few different values of the ratio of end-cap depth to cylinder length. The curve that is closest to a straight line is for the concave end-caps with $m = -0.4$, and the most-curved one is for a sphere. Figure 5 shows the relationship between F and f for the frustum of a cone, with a few different values of ratio (top radius)/(bottom radius).

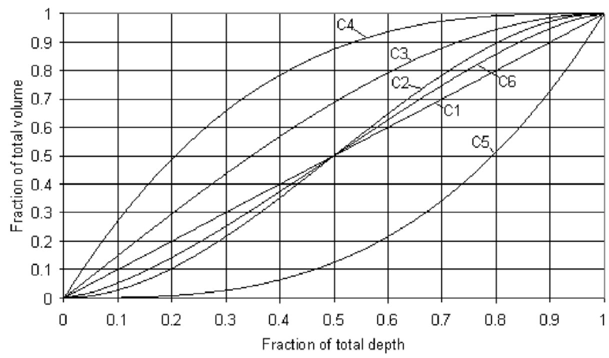

FIGURE 3. Volume-depth relationship $F(f)$ for six container shapes. The six curves are: C4-Cone (point up), C3-hemisphere, C2-sphere, C6-horizontal cylinder, C1-vertical cylinder and C5-cone (point down)

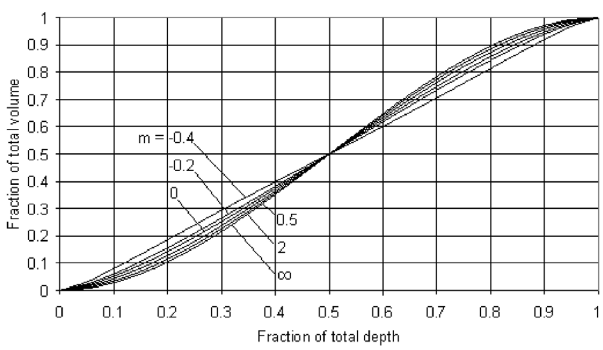

FIGURE 4. Volume-depth relationship $F(f)$ for container shape C7, horizontal cylinders with spherical end-caps, for six values of the ratio m = (end-cap depth)/(cylinder length). Reading top-to-bottom (near $f \approx 0.2$) the curves are respectively for $m = -0.4, -0.2, 0$ (a horizontal cylinder with plane ends), 0.5, 2 and ∞ (a sphere)

CONSTRUCTING A PRACTICAL VOLUME GAUGE

In order to construct a volume gauge, it is more useful to have f as a function of F. In other words, when $F = 0.1$, we need to draw a line on the depth gauge at the corresponding fractional depth f, and mark it "0.1", or the actual volume in litres. The problem is therefore one of finding the *inverse* of the formula giving F as a function of f. Although it would be possible to read this information from the graphs shown above, it may not be sufficiently accurate and a calculation

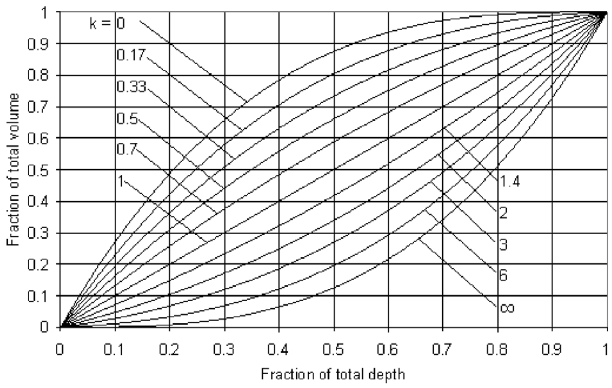

FIGURE 5. Volume-depth relationship $F(f)$ for container shape C8, truncated cones, for eleven values of the ratio k = (top radius)/(bottom radius). The three cases $k = 0, 1$, and ∞ correspond to a cone (point up), vertical cylinder and cone (point down)

is therefore needed. In the first case (a vertical cylinder), it is trivial to invert the relation $F = f$ to produce $f = F$. The cone (point down) has the relation $F = f^3$, which inverts to $f = F^{1/3}$. The cone (point up) is only slightly more difficult, and gives $f = 1 - (1 - F)^{1/3}$. In the case of the frustum of a cone, we can put $y = (1 - k)f$ and rearrange to obtain $1 - F(1 - k^3) = (1 - y)^3$, which can easily be solved for y, and hence for f.

The other cases will have much more complicated analytic solutions (involving the general solution of a cubic equation, and a transcendental equation for the horizontal cylinder), so it will be best to write a computer program to use a numerical method of successive approximation. Generally the problem involves solving the equation $F(f) - G = 0$, where G is one of the specified values (0.1, 0.2, etc.). The Newton-Raphson method can then be applied. In this, we take an approximate value of f, evaluate $F - G$ and its derivative, F', and obtain an improved approximation from the formula $f - (F - G)/F'$. For the horizontal cylinder, with or without end-caps, this process is best applied first to θ (i.e. by making θ the independent variable instead of f), and then finding f in a subsequent step.

When the volume gauge is required to be calibrated in litres (or any other units), and the gauge drawn out full-size, then a screen picture is not sufficient. In this case, the program can be requested to output a table of values giving the selected volumes, the position of the marker (e.g. rounded to 0.1 mm), the length of the line, and the number (if any) to be attached to that marker. This table can then be used for producing an accurate scale by hand, or suitably formatted to be input to an automatic drafting machine.

WRITING THE PROGRAM

If the program is written in Visual Basic (or a similar "visual" language), then the user can enter information into text boxes or to click on option buttons. For example, one set of option buttons can select from the different shapes. For the shapes considered in the present work there would be eight options. Another set of two option buttons can select whether a universal scale is required (i.e. one showing the fractional volume from 0 to 1) or one to suit a real container. Alternatively, these two sets of choices could be provided by drop-down menus.

Other parameters have to be entered into text boxes. These can include: major division, un-numbered main division, un-numbered subdivision, extra parameter (the quantity m as defined for the horizontal cylinder with end caps, and k as defined for the truncated cone. Where actual units are being used, the user would need to supply the scale length (assumed to be in metres), actual container volume and volume units (e.g. litres, gallons, etc.).

A screen-shot illustrating the above ideas is shown in Figure 6. In this, the container is a truncated cone (chosen to illustrate the use of the "extra parameter")

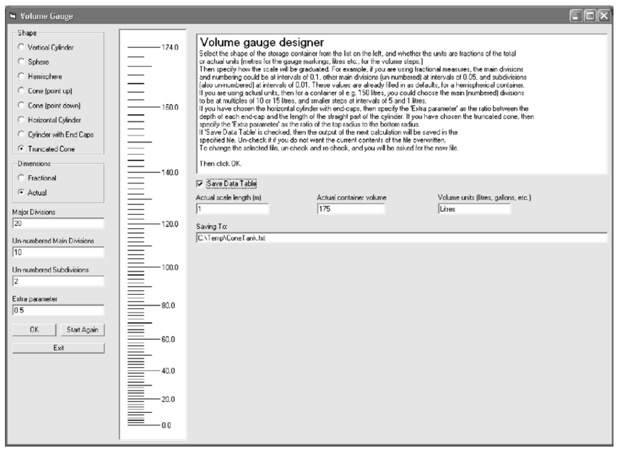

FIGURE 6. A screen shot of the program interface

The text entries shown in Figure 6 have the following meanings.

1. Major division: Since the actual units are being used, and the container volume is 175, it will be convenient to number every 20 units. 10 would also have been a sensible choice, while 5 might give numbering that is too close together. If we had requested values as a fraction of the full scale, then the sensible choices would be 0.05, 0.1 or 0.2.
2. Un-numbered main division: If the 20 units (major division) were only divided into 4 or 5 subdivisions, then there would be no difficulty in reading these. If we divide them into 10 or 20 subdivisions, then some additional marking is necessary to avoid having to count divisions, so an additional longer line is placed on the scale every 10 units.
3. Un-numbered subdivision: The smallest divisions, indicated by the shortest lines, are placed at two units apart. If we had placed these at every unit, then the previous input (un-numbered main division) should have been reduced to 5.

4. Extra parameter. Since, in this example, the container is a truncated cone, the extra parameter, 0.5, is the ratio of the top radius to the bottom radius.
5. Actual scale length. As an example, we suppose that the total height of the container is 1.00 m.
6. Actual container volume. This is specified as 175. Since the smallest divisions are 2 units apart, the largest marked volume in the container will be 174. An additional number is placed on this mark.
7. Volume units (e.g. litres, gallons, etc.). In this example, the units are litres. This information is not used in the calculation, but merely appears as the heading of the output file.
8. Saving to the file where the output is stored, for transfer to a drawing office or an automatic drafting machine, has been selected via the "Save file" dialogue, which is activated when the tick-box "Save data table" is clicked. Normally, the program would first be run without saving the data table. If the picture of the scale is not suitable, then the "Start Again" button would be clicked and any necessary modifications made to the parameters (any that are not changed should stay the same). When the picture of the scale is satisfactory, the "Start Again" button is pressed, and the "Save data table" box is checked. This initiates one more program execution during which the output file is saved. Clicking the tick-box again ensures that the next run does not overwrite the saved data, and clicking it again invites the user to specify a new file for saving.

The vertical picture window shows the output scale, scaled to fit onto the screen. The window to the top right is used for the operating instructions.

The output file can take the form shown in Table 3 (illustrated for the data of figure 6). This is formatted in a suitable form for reading into Excel as a three-column table, plus the heading "Litres". The first column gives the height of the scale mark above the bottom of the container, rounded to 0.1 mm. The second column gives the size of the scale marking: 1 for the shortest lines, 2 for the larger un-numbered divisions, and 3 for the major (numbered) divisions. The third column gives the information that is to be printed alongside each scale marking – the program should output this as a text quantity, so that Excel (for example) does not re-format it. Otherwise there would be a "0" alongside every un-numbered division, whereas we require a blank at these positions. This text file can be edited if, for example, the added value (in this case 174) is too close to the previous value (in this case 160).

TABLE 3. The first and last sections of the output file for the example of Figure 6

```
"Litres"
"Depth","Line-size","Value"
.0,3,"0.0"
.0067,1,""
.0134,1,""
.0202,1,""
.027,1,""
.0339,2,""
.0408,1,""
.0478,1,""
.0548,1,""
.0619,1,""
.069,3,"20.0"
.0762,1,""
.0834,1,""
.0907,1,""
.0981,1,""
.1055,2,""
.1129,1,""
.1204,1,""
.128,1,""
.1357,1,""
.1434,3,"40.0"
.1511,1,""
.159,1,""
.............
.............
.7926,1,""
.8112,1,""
.8304,3,"160.0"
.8502,1,""
.8708,1,""
.8921,1,""
.9142,1,""
.9373,2,""
.9615,1,""
.9868,3,"174.0"
```

Conclusion

We have described a method of graduating a volume scale for reading the contents of containers of various shapes. It should be possible to manufacture the shapes considered to a high accuracy so that a simple specification, such as "horizontal cylinder" is sufficient to define the shape adequately, and the computed volume gauge should be very accurate. There would be several other shapes that can be easily specified in this way, and the calculation can be completed following the techniques used in this paper. More complicated shapes are likely to have less precise manufacturing tolerances. Some shapes require different formulas over different ranges of depth, making it slightly more difficult to use the Newton-Raphson method. In such cases, there may not be much advantage in computing the scale, in comparison with carrying out an "experimental" calibration, by pouring in known volumes and noting the height of the liquid in the container.

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