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Unsteady Magnetoconvective Flow of Bionanofluid with Zero Mass Flux Boundary Condition

(Aliran tak Mantap Magneto-Perolakan bagi Bionanobendalir dengan Keadaan Sempadan Fluks Jisim Sifar)

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ABSTRACT

Induced magnetic field stagnation point flow for unsteady two-dimensional laminar forced convection of water based nanofluid containing microorganisms along a vertical plate has been investigated. We have incorporated zero mass flux boundary condition to get physically realistic results. The boundary layer equations with three independent variables are transformed into a system of ordinary differential equations by using appropriate similarity transformations. The derived equations are then solved numerically by using Maple which use the fourth-fifth order Runge-Kutta-Fehlberg algorithm to solve the system of similarity differential equations. The effects of the governing parameters on the dimensionless velocity, induced magnetic field, temperature, nanoparticle volume fraction, density of motile microorganisms, skin friction coefficient, local Nusselt number and motile density of microorganisms transfer rate are illustrated graphically and tabular form. It is found that the controlling parameters strongly affect the fluid flow and heat transfer characteristics. We compare our numerical results with published results for some limiting cases and found excellent agreement.

Keywords: Forced convection; induced magnetic field; microorganisms; nanofluid; unsteady; zero mass flux

ABSTRAK

Medan magnet teraruh pada titik genangan untuk aliran tak mantap lamina dua dimensi perolakan dipaksa daripada nanobendalir berasaskan air yang mengandungi mikroorganisma bersama plat menegak telah dikaji. Kami telah memasukkan keadaan sempadan fluks jisim sifar untuk mendapatkan keputusan yang realistik. Persamaan lapisan sempadan dengan tiga pembolehubah diubah menjadi sistem persamaan pembezaan biasa dengan menggunakan persamaan transformasi serupa yang sesuai. Persamaan yang telah diperoleh diselesaikan secara berangka dengan menggunakan algorithma Runge-Kutta-Fehlberg keempat-lima dalam Maple untuk menyelesaikan sistem persamaan pembezaan. Kesan parameter pada halaju tak berdimensi, medan magnet teraruh, suhu, pecahan isi padu zarah-zarah nano, ketumpatan mikroorganisma motil, pekali geseran kulit, nombor Nusselt dan ketumpatan motil kadar pemindahan mikroorganisma adalah dipamerkan secara grafik dan dalam bentuk jadual. Didapati bahawa parameter kawalan mempengaruhi aliran bendalir dan ciri-ciri pemindahan haba. Kami membandingkan keputusan berangka ini dengan keputusan yang telah diterbitkan bagi kes terhad dan ia menepati keputusan sedia ada di dalam kajian lepas.

Kata kunci: Keadaan sempadan fluks jisim sifar; medan teraruh magnet; mikroorganisma; bendalir nano; perolakan dipaksa; tak mantap

INTRODUCTION

Nanofluids are a relatively new class of fluids consisting of solid nanoparticles with size less than 100mm within base fluids (Choi 1995; Wong & Leon 2010). The study of nanofluids has gained lots of attention and traction from researchers. Nanofluid plays an important role in many industrial and technology applications since such materials have unique physical and chemical properties. Nanofluids have a wide range of applications in medicine, biomedicine, detergency, power generation in nuclear reactors and more specifically in any heat removal involved industrial applications (Saidur et al. 2011). A complete and thorough study of convective heat transport in nanofluids was made by Buongiorno (2006). Recent research on heat transfer enhancement using nanofluids have been carried by, among others (Kuznetsov & Nield 2014; Mutuku & Makinde 2014; Uddin et al. (2012; Xu & Pop 2014; Yusoff et al. 2014).

Nield and Bejan (2006) defined bioconvection as pattern formation in suspensions of microorganisms, such as bacteria, seaweed and algae which is due to up-swimming of the microorganisms. These microorganisms may include gravitaxis, gyrotaxis or oxytaxis organisms. Bioconvection has a number of applications in biological systems and different aspects of bioconvection have been studied by many researchers. Kuznetsov et al. (2004) conducted a theoretical investigation of a falling bioconvection plume in a deep chamber filled with a fluid saturated porous medium. Kuznetsov and Avramenko (2004) investigated the effects of particles on the stability of bioconvection. The effect of small solid particles in a dilute suspension containing gyrotactic microorganisms has been studied by Geng and Kuznetsov (2005). In this paper, they introduced the idea of effective diffusivity in order to determine the effect of bioconvection on small solid particles. Kuznetsov (2011) showed that the oscillatory mode of nanofluid bioconvection may be caused by the interaction of oxytactic microorganisms, heating or cooling from the bottom and top or bottom-heavy nanoparticle distribution. He used the Galerkin method to solve this problem. Tham et al. (2013), proposed a novel type of nanofluid containing both nanoparticles and motile (gyrotactic) microorganisms. They showed that an addition of motile microorganisms to the suspension improved mass transfer, microscale mixing and improved stability of the nanofluid. Zaimi et al. (2014) developed a model of a bionanofluid toward a stretching and shrinking sheet.

Magnetohydrodynamic (MHD) stagnation point flow of an electrically conducting fluid are frequently used in many practical applications. One of them is metallurgical processes such as drawing, annealing and tinning of copper wires (Ali et al. 2011). Due to diverse applications, many authors investigated the MHD flow in various aspects. Michael (1954) was the first study the two dimensional MHD boundary layer flow problem. Davies (1963) has presented similarity representation of MHD flow in the two dimensional flow past a conducting flat plate. Takhar and Nath (1997) considered unsteady flow on MHD stokes problem for a circular cylinder. They have obtained the new solutions for boundary layer and Navier Stokes equations. Ali et al. (2011) investigated the unsteady MHD mixed convection toward a stagnation flow and mass transfer through viscous incompressible fluid past a vertical surface with induced magnetic field. Numerical solutions are obtained by Keller-box method. They found that the mixed convection parameter show dual solutions for assisting and opposing flow. The effect of radiation and magnetic field on fluid characteristic past an exponentially stretching sheet was studied by Ishak (2011). Sinha and Misra (2014) studied effect of induced magnetic field on MHD stagnation flow and heat transfer on a stretching sheet. Since no work has been carried out on boundary layer flow with consideration of an induced magnetic field in bio-nanofluids, we extend the recent work of Sinha and Misra (2014) to investigate the effect of induced magnetic field, microorganisms and zero mass flux boundary condition. The governing equations are reduced to ODEs by using appropriate similarity transformation before solving numerically. The effects of the physical parameters on the flow, heat, mass and microorganism characteristics are investigated in detail.

MATHEMATICAL FORMULATIONS OF THE PROBLEM

We consider a two-dimensional, unsteady, stagnation point, incompressible viscous electrically conducting fluid with constant properties, hydromagnetic laminar forced convective boundary layer flow of a nanofluid over a solid stationary plate with microorganisms. The effects of the induced magnetic field are taken into account. The electrical field $\vec{E}$ is assumed to be zero. The electrical currents which flow in the fluid will give rise to an induced magnetic field. We assume that the induced magnetic field strength $\bar{H}_e$ acts in the normal direction to the surface, while the normal component of the induced magnetic field $\bar{H}_2$ vanishes when it reaches the wall and the parallel component $\bar{H}_1$ assumed to be $\bar{H}_e$. Let $(\bar{u}, \bar{v})$ and $(\bar{H}_1, \bar{H}_2)$ be the dimensional velocity and dimensional induced magnetic field components along and perpendicular to the plate. The unsteadiness in the flow field is caused by the time dependent magnetic field. The physical configuration of the problem is shown in Figure 1. Here, (i)-(v) represent the momentum, induced magnetic field, thermal, mass diffusion and microorganism boundary layers. Field variables are temperature (T), the nanoparticle volume fraction (C) and the density of motile microorganisms (n). By using these assumptions, the boundary layer equations are shown as follows (Takhar et al. 1993) (Xu & Pop 2014):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\frac{\partial \bar{H}_1}{\partial \bar{x}} + \frac{\partial \bar{H}_2}{\partial \bar{y}} = 0, \tag{2}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial \bar{u}_e}{\partial \bar{t}} + \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0}{\rho}\left(\bar{H}_1\frac{\partial \bar{H}_1}{\partial \bar{x}} + \frac{\partial \bar{H}_1}{\partial \bar{t}} + \bar{H}_2\frac{\partial \bar{H}_1}{\partial \bar{y}}\right) - \frac{\mu_0}{\rho}\left(\bar{H}_e\frac{\partial \bar{H}_e}{\partial \bar{x}} + \frac{\partial \bar{H}_e}{\partial \bar{t}}\right), \tag{3}$$

$$\frac{\partial \bar{H}_1}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{H}_1}{\partial x} + \bar{v}\frac{\partial \bar{H}_1}{\partial \bar{y}} - \bar{H}_1\frac{\partial \bar{u}}{\partial \bar{x}} - H_2\frac{\partial \bar{u}}{\partial \bar{y}} = \alpha_1\frac{\partial^2 \bar{H}_1}{\partial \bar{y}^2}, \tag{4}$$

$$\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} + \tau D_B\frac{\partial T}{\partial \bar{y}}\frac{\partial C}{\partial \bar{y}} + \tau\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial \bar{y}}\right)^2, \tag{5}$$

$$\frac{\partial C}{\partial \bar{t}} + \bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_B\frac{\partial^2 C}{\partial \bar{y}^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{6}$$

$$\frac{\partial n}{\partial \bar{t}} + \bar{u}\frac{\partial n}{\partial \bar{x}} + \bar{v}\frac{\partial n}{\partial \bar{y}} + \frac{\tilde{b}W_c}{C_\infty}\left[\frac{\partial}{\partial \bar{y}}\left(n\frac{\partial C}{\partial \bar{y}}\right)\right] = D_m\left(\frac{\partial^2 n}{\partial \bar{y}^2}\right). \tag{7}$$

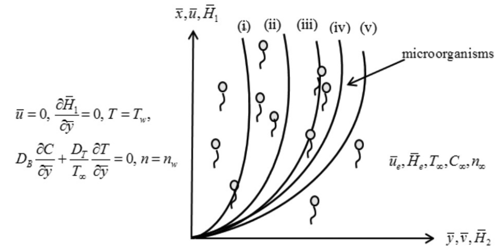


FIGURE 1. Schematic diagram of the problem

The relevant boundary conditions are (Ali et al. 2011; Kuznetsov & Nield 2014)

$$\begin{aligned}
&\bar{u} = 0,\ \bar{v} = 0, \frac{\partial \bar{H}_1}{\partial \bar{y}} = 0,\ \bar{H}_2 = 0,\ T = T_w,\\
&D_B \frac{\partial C}{\partial \bar{y}} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial \bar{y}} = 0,\ n = n_w \ \text{ as } \ \bar{y} = 0,\\
&\bar{u} = \bar{u}_e(\bar{x},\bar{t}),\ \bar{H}_1 \to \bar{H}_e(x,t),\\
&T \to T_\infty,\ C \to C_\infty,\ n \to 0 \ \text{ as } \ \bar{y} \to \infty,
\end{aligned} \tag{8}$$

where u_e: velocity, μ_0: magnetic permeability, $\alpha_1 = 1/\mu_0\sigma$: magnetic diffusivity, σ: electric conductivity, υ: kinematic viscosity, μ: dynamic viscosity, ρ: fluid density, $\tilde{b}$: chemotaxis constant, W_c: maximum cell swimming speed, α: thermal diffusivity of the fluid, $\tau = (\rho c)_p/(\rho c)_f$: ratio of effective heat capacity of the nanoparticle material to the fluid heat capacity, D_B: Brownian diffusion coefficient, D_T: thermophoretic diffusion coefficient, D_m: microorganisms diffusion coefficient. Following Takhar et al. (1993), we adopt the following similarity transformations:

$$\begin{aligned}
&\eta = L^{-1}\left(1-\lambda t^*\right)^{-1/2}\bar{y},\ \ \lambda t^* < 1,\ t^* = \upsilon L^{-2}\bar{t},\\
&\bar{u} = \upsilon L^{-2}\bar{x}\left(1-\lambda t^*\right)^{-1} f'(\eta),\ \ \bar{v} = -\upsilon L^{-1}\left(1-\lambda t^*\right)^{-1/2} f(\eta),\\
&\bar{u}_e = \upsilon L^{-2}\bar{x}\left(1-\lambda t^*\right)^{-1},\\
&\bar{H}_e = a\bar{x}\left(1-\lambda t^*\right)^{-1},\ \ \bar{H}_1 = a\bar{x}\left(1-\lambda t^*\right)^{-1} s'(\eta),\\
&\bar{H}_2 = -aL\left(1-\lambda t^*\right)^{-1/2} s(\eta)\\
&T = \left(T_w - T_\infty\right)\theta(\eta) + T_\infty,\ \ C = C_\infty\phi(\eta) + C_\infty,\\
&n = n_w\chi(\eta).
\end{aligned} \tag{9}$$

Here the dimensionless variables are η (similarity), $f'(\eta)$(velocity),$\theta(\eta)$ (temperature), $\phi(\eta)$(nanoparticle volume fraction), $s'(\eta)$ (induced magnetic field) and $\chi(\eta)$ (microorganisms), t^* is the dimensionless time; a is a constant, λ represents the unsteadiness in the flow field. Subscripts w and ∞ denote at the wall and in the free stream, respectively. L is the characteristic length, and prime denotes ordinary derivative with respect to. By substituting (9) into (3) - (7) the following ordinary differential equations:

$$\begin{aligned}
&f''' + ff'' - f'^2 + \beta_1\left(s'^2 - ss'' - 1\right) + 1 +\\
&\lambda\left[1 + \beta_2\left(\frac{1}{2}\eta s'' + s' - 1\right) - \left(\frac{1}{2}\eta f'' + f'\right)\right] = 0,
\end{aligned} \tag{10}$$

$$\alpha_2 s''' + fs'' - sf'' - \lambda\left(\frac{1}{2}\eta s'' + s'\right) = 0, \tag{11}$$

$$\theta'' + \Pr\left(f\theta' - \frac{1}{2}\lambda\eta\theta' + Nb\theta'\phi' + Nt\theta'^2\right) = 0, \tag{12}$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + Sc\, f\,\phi' - \frac{1}{2}Sc\,\lambda\,\eta\,\phi' = 0, \tag{13}$$

$$\chi'' + Sb\, f\,\chi' - \frac{1}{2}Sb\eta\lambda\chi' - Pe\left[\chi\phi'' + \phi'\chi'\right] = 0. \tag{14}$$

The boundary conditions in (8) become

$$\begin{aligned}
&f(0) = 0,\ f'(0) = 0,\ s''(0) = 0,\ s(0) = 0,\ \theta(0) = 1,\\
&Nb\phi'(0) + \mathrm{Nt}\,\theta'(0) = 0,\ \chi(0) = 1,\ f'(\infty) = 1,\\
&s'(\infty) = 1,\ \theta(\infty) = \phi(\infty) = \chi(\infty) = 0.
\end{aligned} \tag{15}$$

Here, the controlling parameters involved as previously mentioned dimensionless, (10)-(15) are $\beta_1 = \mu_0 a^2/\rho\upsilon^2 L^{-4}$ (Takhar et al. 1993) and $\beta_2 = \mu_0 a/\rho\upsilon^2 L^{-2}$ are the dimensionless magnetic pressure gradient and modified magnetic pressure gradient, respectively, $\alpha_2 = \alpha_1/\upsilon$: reciprocal of the magnetic Prandtl number, $\Pr = \upsilon/\alpha$: Prandtl number, $Nb = \tau D_B C_x/\upsilon$: Brownian motion parameter, $Nt = \tau D_T(T_w - T_\infty)/\upsilon T_\infty$: thermophoresis parameter, $Sc = \upsilon/D_B$: Schmidt number, $Sb = \upsilon/D_m$: bioconvection Schmidt number, and $Pe = \tilde{b}W_c/D_m$: bioconvection Péclet number. It is noted that, we extend the published paper of Sinha and Misra (2014) by adding concentration/nanoparticle volume fraction (6) and microorganism (7). Furthermore, nanoparticle terms (Brownian motion and thermophoresis), zero mass flux boundary conditions and unsteady flow were considered.

PHYSICAL QUANTITIES

The quantities of engineering interest in this study are the local skin friction coefficient $C_{f_{\bar{x}}}$, local Nusselt number $N_{u_{\bar{x}}}$ and the local density number of motile microorganisms defined as:

$$\begin{aligned}
&C_{f_{\bar{x}}} = \frac{\mu}{\rho\bar{u}_e^2}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},\quad Nu_{\bar{x}} = \frac{-\bar{x}}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0},\\
&Nn_{\bar{x}} = \frac{-\bar{x}}{n_w}\left(\frac{\partial n}{\partial\bar{y}}\right)_{\bar{y}=0},
\end{aligned} \tag{16}$$

Substitute (9) into (16) we obtain,

$$\begin{aligned}
&\mathrm{Re}_{\bar{x}}^{1/2} C_{f_{\bar{x}}} = f''(0),\quad \mathrm{Re}_{\bar{x}}^{-1/2} Nu_{\bar{x}} = -\theta'(0),\\
&\mathrm{Re}_{\bar{x}}^{-1/2} Nn_{\bar{x}} = -\chi'(0).
\end{aligned} \tag{17}$$

Here, is the local Reynolds number.

SPECIAL CASE

It is interesting to note that in the absence of the (13) and (14), λ = 0 (steady case), $Nb = Nt = 0$, our problem reduces to Sinha and Misra (2014) when $a = c$ and $Nr = 0$ in their paper.

NUMERICAL SOLUTIONS AND VALIDATION

To validate the accuracy of the present code, the numerical result for the local skin friction coefficient and the local Nusselt number when $\lambda = 0$ (steady case), $\beta_1 = \beta_2 = 0$ (no magnetic field), $Nb = Nt \rightarrow 0$ and absence of (11), (13)-(14) reduce to Kiwan and Al-Nimr (2010). They have solved similarity solution for boundary layer flows under the velocity slip and temperature jump condition. Comparison is also made with previously published results (Kiwan & Al-Nimr 2010) for local skin friction and local Nusselt number as shown in Table 1, where the comparison is found to be in very good agreement. We have not compared our numerical results with Sinha and Misra (2014) because their boundary conditions are different from ours.

TABLE 1. Values of local Nusselt number and skin friction coefficient for Pr = 1

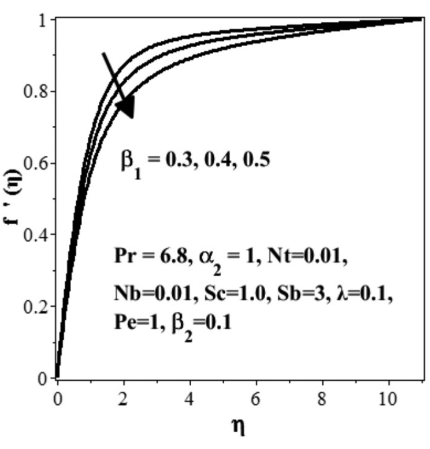
	$-\theta'(0)$	$f''(0)$
Kiwan and Al-Nimr (2010)	1.23259	0.57047
Present study	1.23259	0.57047

RESULTS AND DISCUSSION

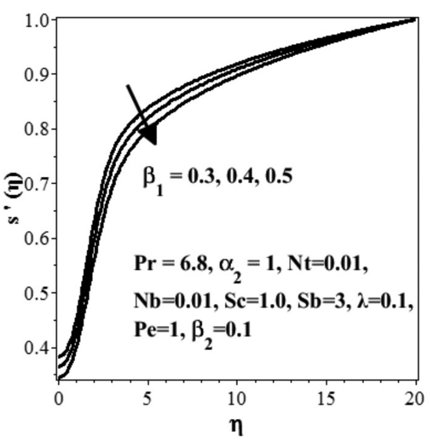
The system of (10)-(14), subject to the boundary conditions in (15) are solved numerically by the Runge Kutta Felhberg fourth-fifth order in Maple 16.0 software. The numerical results are presented in the form of dimensionless velocity, induced magnetic field, temperature, nanoparticles volume fraction and microorganisms. For our numerical calculation, the transformed systems depend on the parameters Pr, λ, *Nb, Nt*, β_1, β_2, α_2, *Sc, Sb, Pe*. However, we do not explicitly examine the influence of Pr, *Pe, Nb* and *Nt* since these parameters have been considered in many previous studies and their effects are well documented (Aziz et al. 2012; Idowu et al. 2013; Mutuku & Makinde 2014). Here we aimed to exhibit the effects of parameters β_1, β_2, α_2, *Sc, Sb* on dimensionless velocity, induced magnetic field, temperature, nanoparticles volume fraction and microorganisms. Since there are no experimental values of the controlling parameters, so we rely on published paper. For skin friction coefficient, heat transfer, microorganism transfer and microorganisms transfer characteristics, we examine the effect of α_2, β_1 and β_2 and and fixed the value of Pr = 6.8 (water based nanofluid) $Nb = Nb = 0.1$, $Sc = Sb = Pe = 1$, $\lambda = 0.1$. For other physical quantities, we choose the value of the controlling parameter range between $1 \leq \alpha_2 \leq 50$, $0 \leq \lambda \leq 1$, $0 \leq \beta_1, \beta_2 \leq 1$, $0 \leq Nb, Nt \leq 0.5$, $1 \leq Sc \leq 3$, $1 \leq Sb \leq 2$ (Aziz et al. (2012; Sinha & Misra 2014; Takhar et al. 1993).

EFFECT OF β_1 AND β_2

Figures 2 and 3 illustrate the effect of the magnetic pressure gradient parameter β_1 and modified pressure gradient parameter β_2 on the dimensionless velocity and induced magnetic field. The effect of β_1 and β_2 on the dimensionless temperature, nanoparticle volume fraction and microorganism are negligible. As we can observe from Figures 2(a) and 3(a), as the magnetic field intensity increase, the velocity profiles decrease. The presence of β_1 and β_2 increases the magnitude of the Lorentz force and retards the magnitude of the velocity. Lorentz force which is a drag-like force that produces more resistance causes a reduction in the fluid velocity. It is noticed from Figures 2(b) and 3(b), the dimensionless induced magnetic field decreases with increases of β_1 and β_2. A similar trend was noted by Glauert (1961) in the case of steady state flow. β_1 and β_2 only exist in the momentum equations and because of the decoupled equations, these parameters will be more significant to velocity profiles and induced magnetic field profiles. There is a less significant effect of β_1 and β_2 on the dimensionless concentration, temperature and microorganism (data not shown).

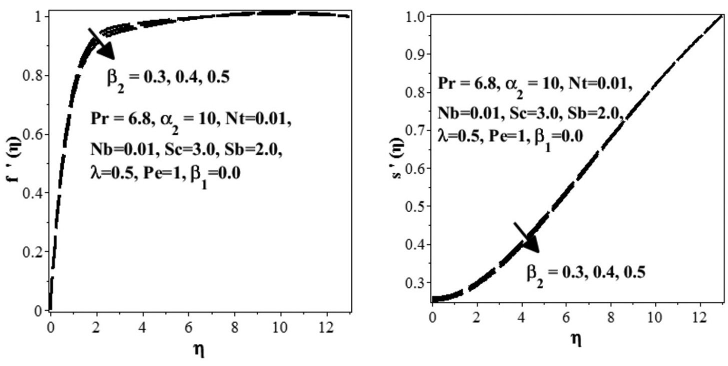


(a)



(b)

FIGURE 2. Effect of β_1 on the (a) dimensionless velocity (b) induced magnetic field



(a) (b)

FIGURE 3. Effect of β_2 on the (a) dimensionless velocity (b) induced magnetic field

EFFECT OF α_2

Figure 4 shows the influences of α_2 on the dimensionless induced magnetic field. We can observed that, as the values of α_2 increase, the induced magnetic field at the wall increases. When α_2 increases, the momentum diffusivity increases. The magnitude of induced magnetic field rises when the magnetic field diffuses in the medium. The effect of α_2 on dimensionless velocity, temperature, nanoparticle volume fraction and microorganism is less significant because α_2 is more pronounced on the induced magnetic field equation. Moreover, α_2 acts as an enhancement of the induced magnetic field equation.

EFFECT OF *Sc*

Figure 5 demonstrates the dimensionless concentration for different values of the Schmidt number *Sc* for nonzero values of the remaining parameters. It is found that at the vicinity of the plate, as *Sc* increases, the dimensionless concentration profiles increase. Physically, higher *Sc* implies either lower solutal diffusivity for a uniform fluid dynamic viscosity or higher dynamic viscosity for uniform solutal diffusivity. There are no significant effect of *Sc* on dimensionless velocity, induced magnetic field, temperature and microorganism.

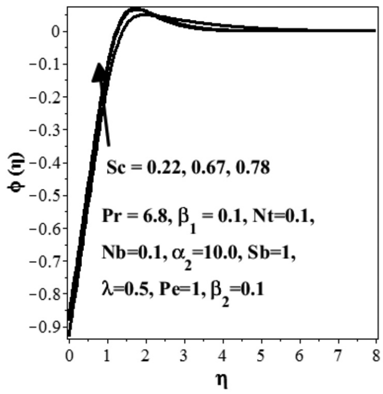


FIGURE 5. Effect of Schmidt number *Sc* on the dimensionless velocity nanoparticle volume fraction

EFFECT OF *Sb*

Figure 6 shows the effect of Sb on the dimensionless density of the motile microorganisms. The dimensionless density of the motile microorganism is strongly influenced by *Sb* because this parameter *Sb* occurs in the equation governing the microorganism (14). It is noted that for a rising *Sb,* the dimensionless density of the motile microorganisms is reduced.

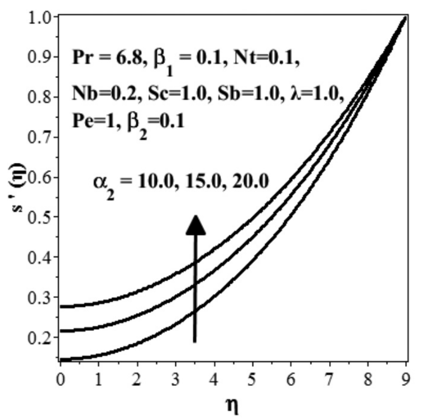


FIGURE 4. Effect of α_2 on the dimensionless induced magnetic field

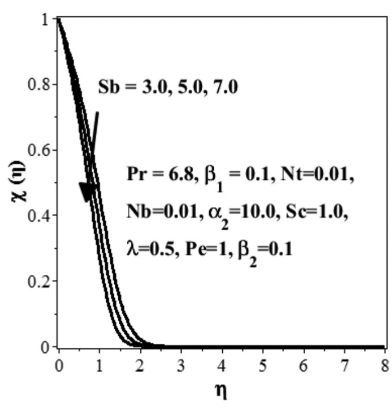


FIGURE 6. Effect of bioconvection Schmidt number *Sb* on the dimensionless microorganism

TABLE 2. Results of the skin friction coefficient, Nusselt number, density of motile microorganism for various α_2, β_1 and β_2 at fixed Pr = 6.8, $Nb = Nb = 0.01$, $Sc = Sb = Pe = 1$, $\lambda = 0.1$

α_2,	β_1	β_2	$f''(0)$	$-\theta'(0)$	$\lvert -\chi'(0) \rvert$
1.0	0.1	0.1	1.3651	0.3258	0.2177
5.0	0.1	0.1	1.3698	0.3287	0.2173
10.0	0.1	0.1	1.3764	0.3324	0.2166
20.0	0.1	0.1	1.3890	0.3395	0.2151
50.0	0.1	0.1	1.4167	0.3547	0.2117
1	0.0	0.1	1.2568	1.0906	0.1103
1	0.25	0.1	1.0654	1.0250	0.0986
1	0.50	0.1	0.8388	0.9349	0.0833
1	0.75	0.1	0.5582	0.7933	0.0603
1	1.0	0.1	0.1838	0.4673	0.0053
1	0.1	0.0	1.2609	1.0982	0.1150
1	0.1	0.25	1.2518	1.0955	0.1141
1	0.1	0.50	1.2426	1.0929	0.1133
1	0.1	0.75	1.2333	1.0902	0.1124
1	0.1	1.0	1.2239	1.0875	0.1116

EFFECT OF $f''(0)$, $-\theta'(0)$ AND $\lvert -\chi'(0) \rvert$

Table 2 exhibits the various parameters on the skin friction coefficient, heat transfer and microorganism transfer. It is clear that an increase in the magnetic Prandtl number leads to an increase in the skin friction coefficient and heat transfer but decreased microorganism transfer rates. Increasing magnetic parameter leads to decreased skin friction coefficient, heat transfer rate and motile microorganism rate.

CONCLUSION

Induced magnetic field stagnation point flow for unsteady two-dimensional laminar forced convection of water based nanofluids containing microorganisms along a vertical plate has been investigated numerically. The momentum, induced magnetic field, temperature, nanoparticle volume fraction and microorganism equations are transformed into a set of ordinary differential equations using appropriate transformations before being solved numerically. Based on the numerical results, we observed that:

$f''(0)$ increases with α_2 but reduces with β_1 and β_2;
$-\theta'(0)$ increases with α_2 but decrease with β_1 and β_2;
$\lvert -\chi'(0) \rvert$ is decreased with increasing α_2 and β_1 and β_2.

This study suggested several future areas for research especially for the non- Newtonian bionanofluid with induced magnetic field. Besides, further research can be carried out for convective boundary layer problem with different geometries (cone, cylinder and sphere).

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