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On Some New Exponential Ratio Estimator of Population Mean in Two Phase Sampling

(Beberapa Penganggar Nisbah Eksponen Baharu bagi Min Populasi dalam Persampelan Dua Fasa)

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ABSTRACT

In this paper, we suggest employing the exponential ratio estimator to estimate the mean of the study variable using a two-phase sample strategy with two modified auxiliary variables. Several researchers discussed the properties of the estimators they proposed and discovered that the estimators in their studies were relatively efficient. The estimators previously studied are listed chronologically in the appendix to this paper. In two phase sampling, the estimator's mean square errors and relative efficiencies are calculated using auxiliary variable information. To assess the properties of our proposed estimator, we noticed that it has a lower mean square error (MSE) than the classical ratio estimator and some other exponential ratio estimators. The estimator is more useful than other estimators in solving real-world issues, notably in engineering, environmental science, management, and biological sciences. The proposed estimator has been applied to real-world data sets such as BRICS, Son's Head Measurement, Number of Hospital Beds, Sale Price of Residence, Ambient Pressure (AP), and Heating Load. In survey research, our suggested estimator has also been demonstrated to be more effective.

Keywords: Auxiliary variable; exponential; mean square error; two phase

ABSTRAK

Dalam makalah ini, kami mencadangkan penggunaan penganggar nisbah eksponen untuk menganggar min pemboleh ubah kajian menggunakan strategi sampel dua fasa dengan dua pemboleh ubah tambahan yang diubah suai. Beberapa penyelidik membincangkan sifat penganggar yang mereka cadangkan dan mendapati bahawa penganggar dalam kajian mereka adalah agak cekap. Penganggar yang dikaji sebelum ini disenaraikan secara kronologi dalam lampiran kertas ini. Dalam persampelan dua fasa, ralat purata kuasa dua penganggar dan kecekapan relatif dihitung menggunakan maklumat pemboleh ubah tambahan. Untuk menilai sifat penganggar yang dicadangkan ini, kami mendapati ia mempunyai ralat min kuasa dua (MSE) yang lebih rendah daripada penganggar nisbah klasik dan beberapa penganggar nisbah eksponen lain. Penganggar ini lebih baik daripada penganggar lain dalam menyelesaikan isu dunia sebenar, terutamanya dalam kejuruteraan, sains alam sekitar, pengurusan dan sains biologi. Penganggar yang dicadangkan telah digunakan pada set data dunia sebenar seperti BRICS, Ukuran Kepala Anak, Bilangan Katil Hospital, Harga Jualan Kediaman, Tekanan Ambien (AP) dan Beban Pemanasan. Dalam kajian tinjauan, penganggar cadangan kami juga telah ditunjukkan sebagai lebih berkesan.

Kata kunci: Eksponen; pemboleh ubah bantu; purata kuasa dua; ralat dua fasa

INTRODUCTION

The auxiliary variable X, which explains the study variable Y, can be used to determine the population mean of 'N' units. In certain cases, auxiliary variable X prior information is not accessible; yet, employing the auxiliary information in double sampling is useful in predicting the population mean. Neyman (1938) encountered a problem determining the variable of interest y when the variable of interest y is costly to measure or information about the auxiliary variable is not readily available for the population, so he measured another relatively inexpensive variable x that is correlated with the variable of interest y.

When certain conditions are met, this sampling design may be favoured: 1). The expense associated with selecting a sample to investigate the variable(s) of interest is high. 2). The absence of a sampling frame, and 3). The population's auxiliary variable information is not easily accessible. Two-phase sampling can be carried out using two distinct methods: 1). When a bigger initial sample encompasses a secondary sample within it, and 2). If the second-phase sample is selected without any dependency on the first-phase sample.

If we do not have auxiliary variable information, we estimate population variables X (auxiliary) in the first phase and then Y (study variable) in the second. The primary sample is the first phase sample selection, which consists of $n_1(n_1 < N)$ units. In the first phase, we choose a sample from 'N' units using the simple random sample without replacement (SRSWR) approach. In some cases, another auxiliary variable (Z) can be used to extract information from the first auxiliary variable (X), and both variables can be seen in the first stage sample. The relevant information for auxiliary variable (X) is computed in the first phase and then sent to the second phase. In the second step, the population mean of research variable Y is calculated using auxiliary variable X. Another sample of units $(n_2 < n_1)$ is drawn for this purpose with SRSWR from the primary sample. A primary sample is one with a sample size of n_1 drawn from the population 'N' and the sample n_2 from n_1 is subsample of our study. This whole mechanism is known as SRS without replacement. To estimate the study variable 'Y,' three common estimation methods are used throughout the literature. These are the ratio, product, and regression methods. For example, Watson (1937) used regression estimation to estimate the average area of plant leaves.

Cochran (1940) suggests using the ratio technique of estimation, if there is a positive correlation between the auxiliary and the study variable. Furthermore, Robson (1957) suggested a product estimation approach in real-world cases where X and Y are negatively associated. Bahl and Tuteja (1991) developed exponential product and ratio type estimators, when negative and positive associations are occurred between study and auxiliary variables respectively. Several authors, including Srivastava (1971), Hidiroglou (2001), Singh and Espejo (2007), Singh and Vishwakarma (2007), Singh, Tailor and Tailor (2010) and Tailor et al. (2012) also provided more accurate estimators for the unknown population mean of the study variable 'Y'. Lu, Yan and Peng (2014) introduced a unique exponential ratio estimator that uses two auxiliary variables to estimate the research variable 'Y' in two phases. Khan, et al. (2014) used stratification in two phase sampling with auxiliary variables to estimate population mean of the variable of interest. Furthermore, Rashid, ul Amin and Hanif (2015) estimated study variables in two phases using an exponential ratio transformation and a product estimator. Jabeen, Sanaullah and Hanif (2015) suggested a generalised version of exponential estimators to estimate the population mean using auxiliary information in two phase sampling. Sanaullah, Hanif and Asghar (2016) presented a more efficient version of the generalised exponential estimator for predicting population mean than the prior ones. Several scholars, including Asghar, Sanaullah and Hanif (2017), and Shabbir and Gupta (2017) propose a generalized variant of the exponential estimator for estimating the mean of the study variable in two-stage sampling using one or more auxiliary variables. Al-Marshadi, Alharby and Shahbaz (2018) developed the method based on the logarithm of auxiliary information for calculating the variance of study variable. Akhlaq, Ismail and Shahbaz (2019) developed a more efficient exponential estimator for analysing process variability utilising auxiliary information than previous estimators. Clement and Inyang (2020) presented a class of stratified ratio estimators in double sampling to estimate the mean for calibration using the co-efficient of kurtosis of the secondary variable. For measuring population variance, Cekim and Kadilar (2020) proposed a ratio estimator based on the ln function. Hassan et al. (2020) were inspired by Cekim and Kadilar (2020) and developed a new estimator for calculating population mean by integrating the exponential and ln ratio functions in double sampling using two ancillary variables. Khan, Gupta and Farhat (2020) proposed using a generalised ratio-cum-regression estimator when the probability sampling is not equal.

In motivation with the given argument for our study, we have recommended an exponential ratio type estimator with transformation and forecasted the MSE. To determine the relative efficiency of our estimator, we examined the performance of other estimators and compared it to ours. When compared to previous estimators, we found that our estimator has the lowest MSE and the highest relative efficiency.

Consider an N-unit population, $T = (T_1, T_2, T_3 ...., T_N)$, with 'Y' and 'X' representing study and auxiliary variables, respectively.

The population means for the study and auxiliary variables are as follows:

$$\bar{Y}=\sum_{i=1}^{N}\left(\frac{y_i}{N}\right) \text{ and } \bar{X}=\sum_{i=1}^{N}\left(\frac{x_i}{N}\right)$$

Sukhatme (1962) was the first who proposed the classical estimator of ratio type in double sampling as:

$$\bar{Y}_r=\left(\frac{\bar{y}}{\bar{x}}\bar{x}'\right);\bar{x}\neq 0 \quad (1)$$

where $\bar{y}$ and $\bar{x}$ are the sample means for the study and auxiliary variables second phase and $\bar{x}'$ is the sample mean of first phase auxiliary variable.

Let $\bar{Y}$ and $\bar{X}$ be the population means, $\bar{y}=\sum_{i=1}^{n}\left(\frac{y_i}{n}\right)$ and $\bar{x}=\sum_{i=1}^{n}\left(\frac{x_i}{n}\right)$ are unbiased estimators of them.

A modification is made by Singh and Vishwakarma (2007) in Bahl and Tuteja (1991) estimator is given as:

$$\bar{Y}_{sv}=\bar{y}_2\, exp\left[\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_1+\bar{x}_2}\right]. \quad (2)$$

where $\bar{y}_2$ and $\bar{x}_2$ are the sample means for the second phase study and auxiliary variables, and $\bar{x}_1$ is the sample mean for the first phase auxiliary variable.

The mean square error of $\bar{Y}_{sv}$ is given as follows

$$MSE\left(\bar{Y}_{sv}\right)=\bar{Y}^2C_y^2\left[f_2+\frac{C_x}{4C_y}\left(f_2-f_1\right)\left(\frac{C_x}{C_y}-\rho_{xy}\right)\right].$$

Singh, Kumar and Smarandache (2008) proposed exponential ratio estimator for estimating study variable ‘Y’ as:

$$\bar{Y}_s=w_0\bar{y}_2+w_2\bar{y}_{rsd}+w_2\bar{y}_{rsed} \quad (4)$$

where

$$\sum\nolimits_{i=0}^{2}w_i=1,\ t_{rsd}=\bar{y}_2\, exp\left[\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_1+\bar{x}_2}\right],$$

$$t_{rsed}=\bar{y}_2\, exp\left[\frac{\left(a\bar{x}_1+b\right)-\left(a\bar{x}_2+b\right)}{\left(a\bar{x}_1+b\right)+\left(a\bar{x}_2+b\right)}\right]$$

$\bar{y}_2$ is the second phase sample study variable mean; $\bar{x}_{(1)}$ is the first phase sample auxiliary variable mean; and $\bar{x}_{(2)}$ is the second phase sample auxiliary variable mean.

Minimum MSE of $\bar{Y}_s$ is

$$min\left(MSE\left(\bar{Y}_s\right)\right)=\bar{Y}^2C_y^2\left(f_2-\left(f_2-f_1\right)\rho_{xy}^2\right) \quad (5)$$

Noor-ul-Amin and Hanif (2012) proposed the estimator in double sampling for estimating study variable as:

$$\bar{Y}_{NH}=\bar{y}_2\, exp\left[\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}-\frac{\bar{X}-\bar{x}_2}{\bar{X}+\bar{x}_2}\right]. \quad (6)$$

where $\bar{y}_2$, $\bar{x}_2$ and $\bar{z}_2$ are the sample means for the study and auxiliary variables of second phase while $\bar{Z}$ and $\bar{X}$ are the sample means of first phase auxiliary variables.

The MSE of the $\bar{Y}_{NH}$ is given as:

$$MSE\left(\bar{Y}_{NH}\right)=\bar{Y}^2\left(\frac{f_2}{4}\left(4C_y^2+C_z^2-4C_yC_z\rho_{zy}\right)-\frac{f_1}{4}\left(C_x^2+4C_yC_x\rho_{xy}-2C_xC_z\rho_{xz}\right)\right). \quad (7)$$

Sanaullah (2012) proposed a modified estimate for two-phase sampling as follows:

$$\bar{Y}_{SA}=\bar{y}_2\, exp\left[\alpha\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}-\left(1-\alpha\right)\frac{\bar{X}-\bar{x}_2}{\bar{X}+\bar{x}_2}\right]. \quad (8)$$

α is positive real number while $\bar{y}_2$, $\bar{x}_2$ and $\bar{z}_2$ are the sample means for the study and auxiliary variables of second phase, $\bar{Z}$ and $\bar{X}$ are the population means of auxiliary variables of the first phase.

Minimum mean square error of $\bar{Y}_{SA}$ is

$$MSE\left(\bar{Y}_{SA}\right)=f_2\bar{Y}^2\left(C_y^2+\hat{\alpha}^2C_z^2-2C_yC_z\rho_{zy}\right)+f_1\bar{Y}^2\left(1-\hat{\alpha}\right)\left(\frac{\left(1-\hat{\alpha}\right)}{4}C_x^2+C_yC_x\rho_{xy}-\hat{\alpha}C_xC_z\rho_{xz}\right). \quad (9)$$

Yadav et al. (2013) motivated from Singh and Vishwakarma (2007) and proposed chain ratio exponential estimator with mean square error as:

$$\bar{Y}_{YUSC} = \bar{y}_2 \, exp\left[\frac{\frac{\bar{x}'}{\left(a\bar{z}'+b\right)}\left(a\bar{Z}+b\right)-\bar{x}}{\frac{\bar{x}'}{\left(a\bar{z}'+b\right)}\left(a\bar{Z}+b\right)+\bar{x}}\right]. \quad (10)$$

$\bar{y}$ and $\bar{x}$ are the sample means for the study and auxiliary variables second phase, $\bar{x}'$ and $\bar{z}'$ are the sample mean of first phase auxiliary variables while a and b are the positive constants.

and

$$MSE\left(\bar{Y}_{YUSC}\right) = \bar{Y}^2\left(f_1 C_y^2 + \frac{1}{4} f_3 C_x^2 - f_3 C_y C_x \rho_{xy} - f_2 \rho_{zy}^2 C_y^2\right). \quad (11)$$

The following is an enhanced version of the exponential ratio estimator proposed by Singh, Choudhury and Kalita (2013) for calculating population mean:

$$\bar{Y}_{SCK} = \bar{y}_2\left[\alpha \, exp\left(\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'}-\bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'}+\bar{x}}\right)+\left(1+\alpha\right)\left(exp\left(\frac{\bar{x}-\bar{x}'\frac{\bar{Z}}{\bar{z}'}}{\bar{x}+\bar{x}'\frac{\bar{Z}}{\bar{z}'}}\right)\right)\right]. \quad (12)$$

$\bar{y}$ and $\bar{x}$ are the sample means for the study and auxiliary variables second phase; $\bar{x}'$ and $\bar{z}'$ are the sample mean of first phase auxiliary variables and α is the positive real number.

Mean square error of $\bar{Y}_{SCK}$ as:

$$MSE\left(\bar{Y}_{SCK}\right) = \bar{Y}^2\left(f_1 C_y^2 - \frac{f_3 C_y C_x \rho_{xy} + f_2 C_y C_z \rho_{zy}}{f_2 C_z^2 + f_3 C_x^2}\right). \quad (13)$$

In continuation of Singh, Choudhury and Kalita (2013) estimator, Kalita, Singh and Choudhury (2013) proposed two exponential ratio estimators for estimating the study variable as:

$$\bar{Y}_{KSC-1} = \bar{y}_2 \, exp\left[\frac{\bar{x}'\frac{\left(a\bar{z}+b\right)}{\left(a\bar{z}'+b\right)}-\bar{x}}{\bar{x}'\frac{\left(a\bar{z}+b\right)}{\left(a\bar{z}'+b\right)}+\bar{x}}\right] \quad (14)$$

where $\bar{x}'$ and $\bar{z}'$ are the sample mean of first phase auxiliary variables; and $\bar{y}$ and $\bar{x}$ are the sample means for the study and auxiliary variables second phase.

a and b are the positive real numbers

MSE of $\bar{Y}_{KSC\text{-}1}$ is given as under

$$MSE\left(\bar{Y}_{KSC-1}\right) = \bar{Y}^2\left(f_1 C_y^2 + \frac{1}{4} f_3 C_x^2 - f_3 C_y C_x \rho_{xy} - f_2 \rho_{xy}^2 C_y^2\right). \quad (15)$$

and

$$\bar{Y}_{KSC-2} = \bar{y}_2 \, exp\left[\frac{\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\frac{\left(a\bar{z}+b\right)}{\left(a\bar{z}'+b\right)}-1}{\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\frac{\left(a\bar{z}+b\right)}{\left(a\bar{z}'+b\right)}+1}\right] \quad (16)$$

MSE of $\bar{Y}_{KSC\text{-}2}$ is given as under

$$MSE\left(\bar{Y}_{KSC-2}\right) = \bar{Y}^2\left(f_1 C_y^2 + 2 f_3 K_{xy}^2 C_x^2 - 2 f_2 K_{zy}^2 C_z^2\right). \quad (17)$$

Singh and Majhi (2014) introduced a chain ratio estimator to estimate the population mean using two auxiliary variables in double sampling:

$$\bar{Y}_{SM} = \bar{y} \, exp\left[\frac{\bar{x}'-\bar{x}}{\bar{x}'+\bar{x}}\right]\left(\frac{\bar{Z}}{\bar{z}'}\right). \quad (18)$$

where the sample means for the study and auxiliary variables second phase are $\bar{y}$ and $\bar{x}$ respectively; $\bar{x}'$ and $\bar{z}'$ are the sample mean of first phase auxiliary variables; and $\bar{Z}$ is the population mean of auxiliary variables of the first phase.

with mean square error of $\bar{Y}_{SM}$ is

$$MSE\left(\bar{Y}_{SM}\right) = \bar{Y}^2\left(f_2 C_y^2 + \frac{1}{4} f_3 C_x^2 - 2 f_2 C_y C_z \rho_{zy} - f_3 C_y C_x \rho_{xy}\right) \quad (19)$$

Exponential estimator of ratio type suggested by Vishwakarma, Kumar and Gangele (2014) for estimating population mean using auxiliary information is given below with MSE:

$$\bar{Y}_{VG} = \bar{y} \, exp\left[\frac{\bar{x}'\frac{\left(a\bar{Z}+\beta\right)}{\left(a\bar{z}'+\beta\right)}-\bar{x}}{\bar{x}'\frac{\left(a\bar{Z}+\beta\right)}{\left(a\bar{z}'+\beta\right)}+\bar{x}}\right] \quad (20)$$

and

$$MSE\left(\bar{Y}_{VG}\right)=\bar{Y}^2\left(f_1C_y^2+\frac{1}{4}f_3C_x^2-f_2C_y^2\rho_{zy}^2-f_3C_yC_x\rho_{xy}\right)(21)$$

Singh and Ahmed (2015) propose an improved version of the exponential relationship estimator that uses two secondary variables in two-stage sampling for the study variable 'Y' with MSE. for:

$$\bar{Y}_{S\&A}=\bar{y}\left[\frac{\sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}-\sqrt{\bar{x}}}{\sqrt{\bar{x}_1\frac{\bar{Z}}{\bar{z}_1}}+\sqrt{\bar{x}}}\right] \qquad (22)$$

and

$$MSE\left(\bar{Y}_{S\&A}\right)=\bar{Y}^2\left[f_2C_y^2+\frac{1}{16}\left(f_3C_x^2+f_2C_z^2\right)+\frac{1}{2}\left(f_3C_xC_y\rho_{xy}+f_2C_zC_y\rho_{zy}\right)\right] \qquad (23)$$

where

$$f_1=\left(\frac{1}{n_1}-\frac{1}{N}\right),f_2=\left(\frac{1}{n_2}-\frac{1}{N}\right),\left(f_2-f_1\right)=\left(\frac{1}{n_2}-\frac{1}{n_1}\right)$$

The coefficients of variation of *X, Y* and *Z* are $C_y=\left(\frac{s_y}{\bar{Y}}\right)$, and $C_x=\left(\frac{s_x}{\bar{X}}\right)$ are $C_z=\left(\frac{s_z}{\bar{Z}}\right)$,

min $MSE\left(\bar{Y}_g^{(1)^{(a)}}\right)=\bar{Y}^2C_y^2\left[f_1\left(\rho_{xy}^2-\rho_{zy}^2\right)+f_2\left(1-\rho_{xy}^2\right)\right]$

$=$ $\rho_{zy}=\left(\frac{s_{zy}}{s_zs_y}\right)$ and $\rho_{xz}=\left(\frac{s_{xz}}{s_xs_z}\right)$ are the correlation coefficient of y with x and y, and x with z, respectively.

These estimators, which are explained above, are useful for evaluating the average of study variable in different situations such as in ratio, product and exponential estimation. The main goal of this work is to improve and characterize the properties of exponential ratio type estimators. Transformed auxiliary variables are used in exponential estimator in the methods section. Comparison section consists of mathematical assessment of our estimator with other estimators which are $\bar{Y}_{sv},\bar{Y}_s,\bar{Y}_{NH},\bar{Y}_{SA},\bar{Y}_{YUSC},\bar{Y}_{SCK},\bar{Y}_{KSC-1},\bar{Y}_{KSC-2},\bar{Y}_{SM},\bar{Y}_{VG}$ and $\bar{Y}_{S\&A}$. Empirical study section explains the empirical evidence regarding our proposed estimator. Conclusion section concludes the study of this paper.

Methods

We get our motivation from the estimator proposed by Singh, Choudhury and Kalita (2013), which is referred to as the ratio cum product exponential estimator that only requires one positive real constant, while we proposed an estimator called the exponential ratio estimator that utilizes two positive real constants and transformed auxiliary variables in two-phase sampling. By employing two positive real constants and transformed auxiliary variables, our suggested estimator may outperform Singh, Choudhury and Kalita (2013) and other estimators in terms of accuracy and precision. While Bandyopadhyay (1980) and Srivenkataramana (1980) proposed transformation $\bar{z}_1^*=\left(N\bar{Z}-n_1\bar{z}_{(1)}\right)\Big/\left(N-n_1\right)$ as an unbiased estimator of $\bar{Z}$.

The proposed estimator is given as:

$$\bar{Y}_g^{(1)^a}=\bar{y}_2\exp\left[\left(\tau_1\frac{\left(\bar{x}_1-\bar{x}_{(1)}^*\right)}{\left(\bar{x}_1+\bar{x}_{(1)}^*\right)}\right)+\left(\tau_2\frac{\left(\bar{z}^*-\bar{z}_1\right)}{\left(\bar{z}^*+\bar{z}_1\right)}\right)\right] \qquad (24)$$

where

$$\bar{x}_{(1)}^*=\frac{n_1\bar{x}_{(1)}-n_2\bar{x}_{(2)}}{n_1-n_2}\ and\ \bar{z}^*=\frac{N\bar{Z}-n_1\bar{z}_{(1)}}{N-n_1}$$

To calculate the MSE of our proposed estimator, we write

$$\bar{y}=\bar{Y}+e_y,\bar{x}=\bar{X}+e_x,\bar{z}_1=\bar{Z}+e_{z_1},\bar{z}_2=\bar{Z}+e_{z_2}$$

Such as

$$\left.\begin{array}{l}E\left(e_y\right)=E\left(e_x\right)=E\left(e_{z_1}\right)=E\left(e_{z_2}\right)=0\\ E\left(e_y^2\right)=f_2\bar{Y}^2C_y^2,E\left(e_z^2\right)=f_1\bar{Z}^2C_z^2,\left(f_2-f_1\right)\bar{X}^2C_x^2\\ E\left(e_ze_y\right)=f_1\bar{Y}\bar{Z}C_yC_z\rho_{zy},E\left(e_y\left(e_{x_1}-e_{x_2}\right)\right)=\left(f_1-f_2\right)\bar{Y}\bar{X}C_yC_x\rho_{xy}\end{array}\right\}$$

writing (24) by using above notations we have

$$\bar{Y}_g^{(1)^a}=\left\{\left(\bar{Y}+e_y\right)exp\left(\tau_1\left(\frac{\frac{-n_2}{n_1-n_2}\left(\bar{X}-\bar{X}-e_{x_2}+e_{x_1}\right)}{\frac{2n_1-n_2}{n_1-n_2}\left(\bar{X}-e_{x_1}\right)-\frac{n_2}{n_1-n_2}\left(\bar{X}-e_{x_2}\right)}\right)\right)exp\left(\tau_2\left(\frac{N\left(\bar{Z}-\bar{z}_1\right)}{N\left(\bar{Z}+\bar{z}_1\right)-2n_1\bar{z}_1}\right)\right)\right\} \qquad (25)$$

or

$$\bar{Y}_g^{(1)^a} = \left\{ \left(\bar{Y}+e_y\right) exp\left(\tau_1 \left(\frac{\frac{-n_2}{n_1-n_2}\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}\left(1+\frac{2n_1-n_2}{n_1-n_2}\frac{e_{x_1}}{2\bar{X}}-\frac{n_2}{n_1-n_2}\frac{e_{x_2}}{2\bar{X}}\right)} \right)\right) \right.$$

$$\left. exp\left(\tau_2\left(\frac{N\left(\bar{Z}-\bar{Z}-e_{z_1}\right)}{N\left(\bar{Z}+\bar{Z}+e_{z_1}\right)-2n_1\left(\bar{Z}+e_{z_1}\right)}\right)\right)\right\} \quad (26)$$

or

$$\bar{Y}_g^{(1)^a} = \left\{ \left(\bar{Y}+e_y\right) exp\left(\tau_1 \left(\frac{\frac{-n_2}{n_1-n_2}\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}\left(1+\frac{2n_1-n_2}{n_1-n_2}\frac{e_{x_1}}{2\bar{X}}-\frac{n_2}{n_1-n_2}\frac{e_{x_2}}{2\bar{X}}\right)} \right)\right) \right.$$

$$\left. exp\left(\tau_2\left(\frac{N\left(-e_{z_1}\right)}{N\left(2\bar{Z}+e_{z_1}\right)-2n_1\left(\bar{Z}+e_{z_1}\right)}\right)\right)\right\} \quad (27)$$

The equation (27) will be expanded series at first degree approximation

$$\bar{Y}_g^{(1)^a} = \left[\left(\bar{Y}+e_y\right)\left(1-\frac{n_2\tau_1}{n_1-n_2}\frac{\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}\right)\left(1-\frac{\tau_2 N e_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right)\right] \quad (28)$$

By multiplying all of the terms of equation (28), we get

$$\bar{Y}_g^{(1)^a} = \left(\bar{Y}+e_y-\frac{n_2\bar{Y}\tau_1}{n_1-n_2}\frac{\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right.$$

$$\left.-\frac{n_2\tau_1}{n_1-n_2}\frac{e_y\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_ye_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right) \quad (29)$$

or

$$\left(\bar{Y}_g^{(1)^a}-\bar{Y}\right) = \left(e_y-\frac{n_2\bar{Y}\tau_1}{n_1-n_2}\frac{\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right.$$

$$\left.-\frac{n_2\tau_1}{n_1-n_2}\frac{e_y\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_ye_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right)$$

On further simplification of above equation up to the first order, we may get as:

$$\left(\bar{Y}_g^{(1)^a}-\bar{Y}\right) = \left(e_y-\frac{n_2\bar{Y}\tau_1}{n_1-n_2}\frac{\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right) \quad (30)$$

squaring and taking expectation on both sides

$$E\left(\bar{Y}_g^{(1)^a}-\bar{Y}\right)^2 = E\left(e_y-\frac{n_2\bar{Y}\tau_1}{n_1-n_2}\frac{\left(e_{x_1}-e_{x_2}\right)}{2\bar{X}}-\frac{\tau_2\bar{Y}Ne_{z_1}}{2\bar{Z}\left(N-n_1\right)}\right)^2 \quad (31)$$

Differentiate the above equation with respect to τ_1, τ_2 and equate to zero.

$$\frac{\partial\left(MSE\right)}{\partial\tau_1}=0, \frac{\partial\left(MSE\right)}{\partial\tau_2}=0$$

The optimum values of τ_1 and τ_2 we can possibly get respectively.

$$\tau_1 = \frac{-2\left(n_1-n_2\right)}{n_2}\frac{C_y\rho_{xy}}{C_x} = \tau_1^*\left(say\right)$$
$$\tau_2 = \frac{2\left(N-n_1\right)}{N}\frac{C_y\rho_{yz}}{C_z} = \tau_2^*\left(say\right) \quad (32)$$

$$MSE\left(\bar{Y}_g^{(1)^a}\right) = \bar{Y}^2\left[f_2C_y^2+\tau_1^2\frac{\left(f_2-f_1\right)C_x^2}{4}\left(\frac{n_2}{n_1-n_2}\right)^2\right.$$
$$+\tau_2^2\frac{f_1C_z^2}{4}\left(\frac{N}{N-n_1}\right)^2-\tau_1\frac{\left(f_2-f_1\right)C_xC_y\rho_{xy}}{4}\left(\frac{n_2}{n_1-n_2}\right)$$
$$\left.-\tau_2\frac{f_1C_zC_y\rho_{zy}}{4}\left(\frac{N}{N-n_1}\right)\right]$$

Using values of τ_1* and τ_2* in the equation given above, the minimum MSE given as:

$$\min MSE\left(\bar{Y}_g^{(1)\left(a\right)}\right) = \bar{Y}^2C_y^2\left[f_1\left(\rho_{xy}^2-\rho_{zy}^2\right)+f_2\left(1-\rho_{xy}^2\right)\right] \quad (33)$$

We, now, extend the estimator (33) to the case of several auxiliary variables. For this, suppose n_1 be chosen at first phase and information on (w_1 + w_2) auxiliary variables $x_{(1)1}$, $x_{(1)2}$,..........$x_{(1)w}$ and $x_{(1)2}$,..........$x_{(1)w}$ is obtained. Let n_2 be the random sample of second phase, which is drawn from the n_1 and information for the variables, $x_{(2)1}$, $x_{(2)2}$,..........$x_{(2)w}$ is obtained. Based upon this information, In two-phase sampling, we propose the following estimate of population mean.

$$\overline{Y}_g^{(1)} = \left[\overline{y}_{(2)j} \left(\exp \sum_{i=1}^{w_1} \tau_{ij} \frac{\left(\overline{X} - \overline{x}^*\right)}{\left(\overline{X} + \overline{x}^*\right)} \right) + \left(\exp \sum_{i=w_1+1}^{w=w_1+w_2} \Omega_{ij} \frac{\left(\overline{x}_{(1)i} - \overline{x}^*_{(1)i}\right)}{\left(\overline{x}_{(1)i} + \overline{x}^*_{(1)i}\right)} \right) \right] \quad (34)$$

where

$$\overline{x}^*_{(1)i} = \frac{n_1 \overline{x}_{(1)_i} - n_2 \overline{x}_{(2)_i}}{n_1 - n_2} \text{ and } \overline{x}^* = \frac{N\overline{X} - n_1 \overline{x}_{(1)_i}}{N - n_1}$$

The optimum values of π_i and Ω_i are

$$\pi_i = (-1)^{i+1} \frac{-2(N - n_1)}{n_1} C_{w_1} C_y \frac{\left|R_{yx_i}\right|_{yx_{w_1}}}{|R|_{x_{w_1}}}, (i = 1, 2.......w_1)$$

and

$$\Omega_i = (-1)^{i+1} \left(1 - \frac{n_1}{n_2}\right) \frac{2C_y}{C_{x_i}} \frac{\left|R_{yx_i}\right|_{yx_w}}{|R|_{x_w}}, (i = 1, 2.......w)$$

where $|R|_{x_{w_1}}$ is determinant of correlation matrix of auxiliary variables *X*, $|R|_{x_w}$ is determinant of correlation matrix of auxiliary variables *X*.

COMPARISON OF $\overline{Y}_g^{(1)^a}$ WITH ALREADY DEVELOPED ESTIMATORS

Equation (31) and equation (33) explain the MSE of our proposed estimator. We compared the estimators provided by $\overline{Y}_{sv}, \overline{Y}_s, \overline{Y}_{NH}$ and $\overline{Y}_{SA}$ with our proposed estimator. In comparison to other estimators, we find that the suggested estimator is relatively efficient and has the minimum mean squared error.

i. $MSE\left(\hat{\overline{Y}}_{sv}\right) \geq MSE\left(\overline{Y}_g^{(1)^a}\right)$

$\hat{\overline{Y}}_1$ is more effricent than $\hat{\overline{Y}}_{sv}$ (36)

iff

$$f_1\left(\rho^2_{xy} + \frac{K^2}{4} - K\rho_{xy}\right) - f_2\left(\left(\rho_{xy}\right)^2 + \left(\rho_{yz}\right)^2 + \frac{(K)^2}{4} - K\rho_{xy}\right) \geq 0$$

where $K = C_x / 4C_y$

ii. $MSE\left(\hat{\overline{Y}}_s\right) \geq MSE\left(\overline{Y}_g^{(1)^a}\right) => f_1\left(\rho_{yz}\right)^2 \geq 0$ (37)

iii. $MSE\left(\hat{\overline{Y}}_{NH}\right) \geq MSE\left(\overline{Y}_g^{(1)^a}\right)$

iff

$$f_1\left(\frac{C_x^2}{4} + C_x C_y \rho_{xy} - \frac{1}{2} C_x C_z \rho_{xz} - C_y^2\left(\rho_{xy}^2 - \rho_{yz}^2\right)\right) + f_2\left(\frac{C_z^2}{4} - C_y C_z \rho_{yz} + C_y^2 \rho_{xy}^2\right) \geq 0 \quad (38)$$

iv. $MSE\left(\hat{\overline{Y}}_{SA}\right) \geq MSE\left(\overline{Y}_g^{(1)^a}\right)$

$$f_1(1-\hat{\alpha})\left(\frac{(1-\hat{\alpha})}{4} C_x^2 + C_y C_x \rho_{xy} - \hat{\alpha} C_x C_z \rho_{xz} - C_y^2\left(\rho_{xy}^2 - \rho_{yz}^2\right)\right) + f_2\left(\hat{\alpha} C_z^2 - 2C_y C_z \rho_{yz} + C_y^2 \rho_{xy}^2\right) \geq 0 \quad (39)$$

The proposed estimator $\overline{Y}_g^{(1)^a}$ is considered to be more efficient if the conditions above in (36), (37), (38) and (39) are met.

SIMULATION RESULTS

The simulation investigation was guided by randomly generating populations from a bivariate normal distribution. In this simulation, a random population of 50000 was generated using a conventional bivariate normal distribution for the auxiliary variables X and Z. The study variable Y was derived using these auxiliary variables as $Y_i = X_i + Z_i + e_i$ where e_i equals $N(0,1)$. Two phase samples were generated from this population by simulating 25% of 50000 and 30% of 12500 $(n_1 = 12500, n_2 = 3750)$ first phase and second phase samples, respectively. For each sample, many estimators were computed. The process was repeated 50000 times, and the MSE of each estimator was calculated using 50000 estimates. The results are presented in Table 1.

EMPIRICAL STUDY

To assess the performance of the proposed estimator, we provided empirical and comparative data in this section. For empirical study, we have selected some real population, available in literature. The data of BRICS, Son's Head Measurement, Number of beds in hospital, Sale price of residence, Ambient Pressure (AP) in the limit of 992.89-1033.30 milibar and Heating Load has been used. The descriptions of study variable 'Y' and auxiliary variables 'X' and 'Z' are given in Table 2. The descriptive statistics of the population information is given in Table 3. Table 4 displays the mean squared errors (MSE's) and relative efficiencies (RE's) values for some of the already developed estimators and proposed by us.

Table 1 demonstrates that for (n_1 = 12500, n_2 = 3750), the simulated mean square error of our proposed estimator is less than the mean square error of other estimators used in the study, because the estimator with the lowest mean square error is deemed the most effective, it is an efficient estimator.

TABLE 1. Simulation population

Estimators	Simulation $n_1 = 12500$ $n_2 = 3750$
$MSE\left(\bar{Y}_g^{(1)^{**}}\right)$	0.0005320166
$MSE\left(\bar{Y}_{sv}\right)$	0.0019127328
$MSE\left(\bar{Y}_s\right)$	0.0012416856
$MSE\left(\bar{Y}_{sck}\right)$	0.0087537459
$MSE\left(\bar{Y}_{sm}\right)$	0.0008196766
$MSE\left(\bar{Y}_{NH}\right)$	0.0005583624
$MSE\left(\bar{Y}_{SA}\right)$	0.0012416856

TABLE 2. Depiction of variables for all population

Pop.	X	Y	Z	Populations Sources
1	World Stock Exchange	BRICS Stock Exchange	Chinese Stock Exchange	Chinese,World and BRICS Stock Exchanges
2	1st son head measurement	2nd son head measurement	1st son head breadth	Anderson page# 109, (1958)
3	Active physicians' total number	Number of beds in hospital	Complete Population	Nachtshemim, Neter and Kutner, page 1350, (2004)
4	Completed area of house	Sale price of residence	Number of beds in residence	Nachtshemim Neter & Kutner, page #1353, (2004)
5	Electrical energy production hourly	Ambient Pressure (AP) in the limit of 992.89-1033.30 milibar	Relative Humidity (RH) of ranging 25.56% to 100.16%	Faculty of Engineering, Namık Kemal University, Turkey
6	Wall Area	Heating Load	Relative Compactness	Oxford Centre for Industrial and Applied Mathematics, University of Oxford, UK

TABLE 3. Population means, coefficients of variation, and coefficients of correlation

Pop.	N	n_1	n_2	$\bar{X}$	$\bar{Y}$	$\bar{Z}$	C_y	C_x	C_z	ρ_{xy}	ρ_{yz}	ρ_{xz}
1	274	89	21	0.0004	0.000737	0.0009368	12.1725	9.94814	11.8189	0.41430	0.82962	0.23332
2	25	10	7	185.72	183.84	151.12	0.0535	0.0515	0.0478	0.8124	0.8329	0.8641
3	440	88	18	987.9	1458.67	393010.9	1.5676	1.8094	1.5299	0.9504	0.9237	0.9402
4	522	104	21	2260.6	277894.1	3.471264	0.4958	0.3142	0.2919	0.8194	0.4133	0.5578
5	9568	1000	700	1012.9	454.22	72.639	0.0378	0.0060	0.2046	0.5168	0.3934	0.0956
6	768	300	220	0.7641	22.3071	318.5	0.4520	0.1383	0.1368	0.6222	0.4556	-0.2037

The formula for calculating REs in percentage of previous estimators and our proposed estimator $\bar{Y}_g^{(1)^a}$ w.r.t estimator $\bar{Y}$ are given as:

$$PRE(.,\bar{y}) = \frac{MSE(\bar{y})}{MSE(.)} \times 100$$

$$where(.) = \begin{pmatrix} \bar{Y}_{sv}, \bar{Y}_s, \bar{Y}_{NH}, \bar{Y}_{SA}, \bar{Y}_{YUSC}, \bar{Y}_{SCK}, \bar{Y}_{KSC-1}, \\ \bar{Y}_{KSC-2}, \bar{Y}_{SM}, \bar{Y}_{VG}, \bar{Y}_{S\&A} \text{ and } \bar{Y}_g^{(1)^a} \end{pmatrix}$$

Table 4 provides the exponential mean estimators employing two-phase sampling in chronological order. It demonstrates that the majority of the available estimators produce more efficient results than the classic ratio estimator.

In order to invigorate our study in the field of finance, we sorted out the stock exchange data for the World, Chinese economy and for the BRICS countries. The proposed estimator is justifiable in the field of Finance, which deals with big data science and to prognosticate the uncertain factors influencing the financial performance of exogenous variables and to reduce the likelihood errors in the endogenous variables. To understand this epistemology, we performed the BRICS Stock Exchange in the form of dependent variable and follow the World Stock Exchange and the Chinese as independent variables. After governing this catalytic approach, we found that the MSE for these observed variables is lower which augments the importance of our study estimator.

When the MSEs and REs of the estimators are examined, we find that our proposed estimator outperforms the conventional and already developed ratio estimators for all populations (Table 4). The choice of our estimator is solely rely on availability of the population parameter(s) used in practical fieldwork.

TABLE 4. Populations MSE and PREs

Estimators	I		II		III		IV		V		VI	
	MSE	RE	MSE	RE	MSE	RE	MSE	RE	MSE	RE	MSE	RE
$\hat{\bar{Y}}$	0.000000071	100.00	9.9539	100.00	278574.6	100.00	867742843	100.00	0.3911	100.00	0.3297	100.00
$\hat{\bar{Y}}_{SV}$	0.000000077	*	9.4999	104.77	271965.4	102.43	846511651.1	102.50	0.3893	100.46	0.3268	100.91
$\hat{\bar{Y}}_{S}$	0.000000067	105.88	7.2169	137.92	69852.3	398.80	383197138.1	226.44	0.3573	109.46	0.2820	116.91
$\hat{\bar{Y}}_{NH}$	0.000000028	254.95	9.6642	102.99	136562.2	203.99	817879358.8	106.09	2.4319	*	0.3379	*
$\hat{\bar{Y}}_{SA}$	0.000000029	246.23	3.2067	310.40	57208.8	486.94	858135188.1	101.11	0.3510	111.42	0.2818	117.00
$\hat{\bar{Y}}_{YUSC}$	0.000000077	*	3.6437	273.18	61497.3	452.98	540485964.9	160.54	0.3893	100.46	0.1174	280.72
$\hat{\bar{Y}}_{SCK}$	0.000000092	*	3.2052	310.54	29893.1	931.90	373289693.4	232.45	0.4025	*	0.1418	232.40
$\hat{\bar{Y}}_{KSC-1}$	0.000000067	105.88	3.8398	259.22	59116.6	471.22	467289784.8	185.69	0.3573	109.46	0.0582	566.13
$\hat{\bar{Y}}_{KSC-2}$	0.000000061	117.85	7.3720	135.02	614906.4	*	1786885284	*	2.4319	*	0.1650	199.82
$\hat{\bar{Y}}_{SM}$	0.000000028	254.95	3.6649	271.59	47917.3	581.36	385176725.6	225.28	2.4099	*	0.1331	247.59
$\hat{\bar{Y}}_{VG}$	0.000000029	246.23	3.6437	273.18	61497.3	452.98	540485964.9	160.54	0.3510	111.42	0.1174	280.72
$\hat{\bar{Y}}_{S\&A}$	0.000000028	256.78	14.265	*	448809.4	*	1094178484	*	3.8517	*	0.2436	135.35
$\hat{\bar{Y}}_{g}^{(1)^a}$	**0.000000020**	**354.74**	**3.1889**	**312.1419**	**29295.9**	**950.89**	**358222654**	**242.23**	**0.1404**	**278.5099**	**0.0532**	**619.36**

* Relative Efficiency is less than 100

Conclusion

This work investigates the ratio type of exponential estimators with two auxiliary variables in two phase sampling. The primary goal of the study is to compare the suggested estimator's efficiency with the various existing exponential estimators in the literature. The effectiveness of our proposed estimator is tested using several population datasets from various domains as well as simulated data. When the estimated values of MSE are compared, our proposed estimators have the lowest MSE across all populations and simulation experiment. As a result, our suggested estimator is deemed to be more efficient, while Singh, Choudhury and Kalita (2013) ratio estimator is regarded as the second most efficient estimator in terms of MSE values. Because of the proposed estimator's dominance nature, it may be suggested for practical applications. It is also recommended for future research that the multivariate exponential estimator in two phase sampling with transformed auxiliary variables to estimate the study variable 'Y' can be more helpful than existing estimators.

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APPENDIX

Nomenclature section

Symbols	Description
$\bar{y}$	The sample mean that correspond to the population mean of .
$\bar{x}, \bar{x}', \bar{x}_{(1)}$	The sample means that correspond to the population mean of.
$\bar{z}$	The sample means that correspond to the population mean of.
C_x, C_y, C_z	Coefficient of variation of x, y and z
$\rho_{xy}, \rho_{yz}, \rho_{xz}$	Correlation coefficients between of respectively
n_1, n'	Samples sizes of phase one.
n, n_2	Samples size of phase two.
$\beta_{2(z)}$	Kurtosis
N	Complete population
σ	Standard deviation

$$f_1 = \left(\frac{1}{n'} - \frac{1}{N}\right),\ f_2 = \left(\frac{1}{n} - \frac{1}{N}\right),\ \left(f_2 - f_1\right) = f_3 = \left(\frac{1}{n} - \frac{1}{n'}\right)$$

The Ascending Order of exponential estimators in double sampling

S#	Estimator	Reference	Mean Square Error
1	$\bar{y}$	Unbiased sample mean	$MSE(\bar{y}) = \bar{Y}^2 \frac{1-f}{n} C_y^2$
2	$\bar{Y}_{sv} = \bar{y}_2 \exp\left[\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right]$	Singh and Vishwakarma (2007)	$MSE(\bar{Y}_{sv}) = \bar{Y}^2 C_y^{\,2}\left[f_2 + \frac{C_x}{4C_y}(f_2 - f_1)\left(\frac{C_x}{C_y} - \rho_{xy}\right)\right]$
3	$\bar{Y}_s = w_{0d}\bar{y}_2 + w_{1d}\bar{y}_{rsd} + w_{2d}\bar{y}_{rsed}$	Singh et al. (2008)	$MSE(\bar{Y}_s) = \bar{Y}^2 C_y^{\,2}\left(f_2 - (f_2 - f_1)\rho_{xy}^{\,2}\right)$
4	$\bar{Y}_{NH} = \bar{y}_2 \exp\left[\frac{\bar{Z} - \bar{z}_2}{\bar{Z} + \bar{z}_2} - \frac{\bar{X} - \bar{x}_2}{\bar{X} + \bar{x}_2}\right]$	Noor-ul-Amin and Hanif (2012)	$MSE(\bar{Y}_{NH}) = \bar{Y}^2 \begin{bmatrix} \frac{f_2}{4}\left(4C_y^{\,2} + C_z^{\,2} - 4C_yC_z\rho_{yz}\right) + \\ \frac{f_1}{4}\left(C_x^{\,2} + 4C_yC_x\rho_{yx} - 2C_xC_z\rho_{xz}\right) \end{bmatrix}$
5	$\bar{Y}_{SA} = \bar{y}_2 \exp\left[\alpha\frac{\bar{Z} - \bar{z}_2}{\bar{Z} + \bar{z}_2} - (1-\alpha)\frac{\bar{X} - \bar{x}_2}{\bar{X} + \bar{x}_2}\right]$	Sanaullah et al. (2012)	$MSE(\bar{Y}_{SA}) = \begin{bmatrix} f_2\bar{Y}^2\left[C_y^{\,2} + \hat{\alpha}^2 C_z^{\,2} - 2C_yC_z\rho_{yz}\right] + \\ f_1\bar{Y}^2(1-\hat{\alpha})\begin{bmatrix}\left(\frac{1-\hat{\alpha}}{4}\right)C_x^{\,2} + \\ C_yC_x\rho_{yx} - \hat{\alpha}C_xC_z\rho_{xz}\end{bmatrix} \end{bmatrix}$
6	$\bar{Y}_{YUSC} = \bar{y} \exp\left[\frac{\frac{\bar{x}'}{(a\bar{z}'+b)}(aZ+b) - \bar{x}}{\frac{\bar{x}'}{(a\bar{z}'+b)}(aZ+b) + \bar{x}}\right]$	Yadav et al. (2013)	$MSE(\bar{Y}_{YUSC}) = \bar{Y}^2\left[f_1C_y^2 + \frac{1}{4}f_3C_x^2 - f_3C_xC_y\rho_{xy} - f_2\rho_{zy}^2C_y^2\right]$
7	$\bar{Y}_{SCK} = \bar{y}\begin{bmatrix} \alpha\exp\left(\frac{\bar{x}'\frac{Z}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{Z}{\bar{z}'} + \bar{x}}\right) + \\ (1-\alpha)\exp\left(\frac{\bar{x} - \bar{x}'\frac{Z}{\bar{z}'}}{\bar{x} + \bar{x}'\frac{Z}{\bar{z}'}}\right) \end{bmatrix}$	Singh, Choudhury & Kalita (2013)	$MSE(\bar{Y}_{SCK}) = \bar{Y}^2\left[f_1C_y^2 - \frac{\left(f_3C_xC_y\rho_{xy} + f_2C_zC_y\rho_{zy}\right)^2}{f_2C_z^2 + f_3C_x^2}\right]$
8	$\bar{Y}_{KSC-1} = \bar{y}\exp\left[\frac{\bar{x}'\frac{(a\bar{z}+b)}{(a\bar{z}'+b)} - \bar{x}}{\bar{x}'\frac{(a\bar{z}+b)}{(a\bar{z}'+b)} + \bar{x}}\right]$	Singh, Choudhury & Kalita (2013)	$MSE(\bar{Y}_{KSC-1}) = \bar{Y}^2\left[f_1C_y^2 + \frac{1}{4}f_3C_x^2 - f_3C_xC_y\rho_{xy} - f_2\rho_{xy}^2C_y^2\right]$

9	$\overline{Y}_{KSC-2} = \overline{y}\exp\left[\dfrac{\left(\dfrac{\overline{x}'}{\overline{x}}\right)^{\alpha}\dfrac{(a\overline{z}+b)}{(a\overline{z}'+b)}-1}{\left(\dfrac{\overline{x}'}{\overline{x}}\right)^{\alpha}\dfrac{(a\overline{z}+b)}{(a\overline{z}'+b)}+1}\right]$	Singh, Choudhury & Kalita (2013)	$MSE\left(\overline{Y}_{KSC-2}\right) = \overline{Y}^2\left[f_1C_y^2 + 2f_3K_{yx}^2C_x^2 - 2f_2K_{yz}^2C_z^2\right]$
10	$\overline{Y}_{SM} = \overline{y}\exp\left[\dfrac{\overline{x}'-\overline{x}}{\overline{x}'+\overline{x}}\right]\left(\dfrac{\overline{Z}}{\overline{z}'}\right)$	Singh & Majhi (2014)	$MSE\left(\overline{Y}_{SM}\right) = \overline{Y}^2\left[f_2C_z^2 + \dfrac{1}{4}f_3C_x^2 + f_1C_y^2 - 2f_2C_zC_y\rho_{zy} - f_3C_xC_y\rho_{xy}\right]$
11	$\overline{Y}_{VG} = \overline{y}\exp\left[\dfrac{\overline{x}'\dfrac{(a\overline{Z}+\beta)}{(a\overline{z}'+\beta)}-\overline{x}}{\overline{x}'\dfrac{(a\overline{Z}+\beta)}{(a\overline{z}'+\beta)}+\overline{x}}\right]$	Vishwakarma et al. (2014)	$MSE\left(\overline{Y}_{VG}\right) = \overline{Y}^2\left[f_1C_y^2 + \dfrac{1}{4}f_3C_x^2 - f_2\rho_{zy}^2C_y^2 - f_3C_xC_y\rho_{xy}\right]$
12	$\overline{Y}_{S\&A} = \overline{y}\left[\alpha\exp\left(\dfrac{\sqrt{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}-\sqrt{\overline{x}'}}{\sqrt{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}+\sqrt{\overline{x}'}}\right)\right]$	Singh & Ahmed (2015)	$MSE\left(\overline{Y}_{S\&A}\right) = \overline{Y}^2\left[f_2C_z^2 + \dfrac{1}{16}\left(f_3C_x^2 + f_2C_z^2\right) + \dfrac{1}{2}\left(f_3C_xC_y\rho_{xy} + f_2C_zC_y\rho_{zy}\right)\right]$
13	$\overline{Y}_g^{(1)^a} = \overline{y}_2\exp\left[\left(\tau_1\dfrac{\left(\overline{x}_1-\overline{x}_{(1)}^*\right)}{\left(\overline{x}_1+\overline{x}_{(1)}^*\right)}\right)+\left(\tau_2\dfrac{\left(\overline{z}^*-\overline{z}_1\right)}{\left(\overline{z}^*+\overline{z}_1\right)}\right)\right]$	Our Proposed Estimator	$MSE\left(\hat{\overline{Y}}_1\right) \cong \overline{Y}^2C_y{}^2\left[f_2\left(1-\rho_{xy}{}^2\right) + f_1\left(\rho_{xy}{}^2 - \rho_{yz}{}^2\right)\right]$