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A Comparative Analysis of Stratified Double Folded Ranked Set Sampling Performance Across Various Distributions

(Analisis Perbandingan Prestasi Persampelan Set Berperingkat Berlipat Ganda Berstrata Melalui Pelbagai Taburan)

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ABSTRACT

Efficient statistical estimation is crucial for accurate population parameter estimation. This study introduces and evaluates Stratified Double Folded Ranked Set Sampling (SDFRSS), a modified sampling technique designed to enhance estimation efficiency across various probability distributions. Using Monte Carlo simulations, SDFRSS is compared with Stratified Simple Random Sampling (SSRS), Stratified Ranked Set Sampling (SRSS), and Stratified Median Ranked Set Sampling (SMRSS) based on Mean Squared Error (MSE) and Relative Efficiency (RE) under multiple distributions, including *Normal*, *Student's t*, *Uniform*, *Exponential*, *Geometric*, *Gamma*, *Beta*, *Weibull*, *Log-Normal*, *Logistic*, and *Chi-Square*. The results showed that SDFRSS consistently outperforms SSRS, SRSS, and SMRSS, particularly in skewed and heavy-tailed distributions, by achieving lower MSE and higher efficiency. It effectively reduces estimation errors while maintaining robustness across different sample sizes and stratification structures. However, for some symmetric distributions, SDFRSS does not always yield the lowest MSE, emphasizing the need for distribution-specific selection of sampling methods. Despite increased computational complexity, SDFRSS provides significant gains in precision and efficiency, making it a valuable tool for researchers in fields requiring accurate statistical estimation. Future research should explore its application in high-dimensional data and real-world statistical problems to further establish its practical utility.

Keywords: Stratified Double Folded Ranked Set Sampling; Stratified Median Ranked Set Sampling; Stratified Ranked Set Sampling; Stratified Simple Random Sampling

ABSTRAK

Anggaran statistik yang cekap adalah penting untuk anggaran parameter populasi yang tepat. Kajian ini memperkenal dan menilai Persampelan Set Berperingkat Berlipat Ganda Berstrata (SDFRSS), teknik persampelan terubah suai yang direka untuk meningkatkan kecekapan anggaran merentas pelbagai taburan kebarangkalian. Menggunakan simulasi Monte Carlo, SDFRSS dibandingkan dengan Persampelan Rawak Mudah Berstrata (SSRS), Persampelan Set Peringkat Berstrata (SRSS) dan Persampelan Set Peringkat Median Berstrata (SMRSS) berdasarkan Ralat Purata Kuasa Dua (MSE) dan Kecekapan Relatif (RE) di bawah berbilang pengagihan, termasuk *Normal, t Pelajar, Seragam, Eksponen, Geometri, Gamma, Beta, Weibull, Log-Normal, Logistik* dan *Khi Kuasa Dua*. Keputusan ini menunjukkan bahawa SDFRSS secara tekal mengatasi prestasi SSRS, SRSS dan SMRSS, terutamanya dalam pengedaran condong dan berat, dengan mencapai MSE yang lebih rendah dan kecekapan yang lebih tinggi. Ia berkesan mengurangkan ralat anggaran sambil mengekalkan keteguhan melalui saiz sampel yang berbeza dan struktur stratifikasi. Walau bagaimanapun, untuk sesetengah taburan simetri, SDFRSS tidak selalu menghasilkan MSE terendah, menekankan keperluan untuk pemilihan kaedah pensampelan khusus pengedaran. Walaupun kerumitan pengiraan meningkat, SDFRSS memberikan keputusan yang lebih baik dalam ketepatan dan kecekapan, menjadikannya alatan penting untuk penyelidik dalam bidang yang memerlukan anggaran statistik yang tepat. Penyelidikan masa depan harus meneroka pengaplikasiannya dalam data berdimensi tinggi dan masalah statistik dunia nyata untuk terus mewujudkan utiliti praktikalnya.

Kata kunci: Persampelan Set Kedudukan Berlipat Ganda Berstrata; Persampelan Set Kedudukan Median Berstrata; Persampelan Set Kedudukan Berstrata; Persampelan Rawak Mudah Berstrata

INTRODUCTION

Ranked Set Sampling (RSS) is a statistical method that has experienced considerable evolution since its original development. First introduced by McIntyre in 1952, RSS was devised to enhance the efficiency of estimating mean pasture yields compared to conventional sampling techniques. Since its inception, numerous adaptations and refinements have been applied to the method, establishing RSS as a valuable tool for estimating population parameters. These advancements have extended the scope of RSS beyond its initial agricultural application to various other fields, facilitating more precise and efficient estimation processes. This essay delves into the evolution of RSS, highlighting significant advancements such as Stratified Ranked Set Sampling (SRSS), Extreme Ranked Set Sampling (ERSS), and Folded Ranked Set Sampling (FRSS), culminating in the development of SDFRSS.

McIntyre's (1952) pioneering work on RSS provided a mechanism for estimating the mean of pasture yields using a combination of visual ranking and random selection. This approach enabled a more efficient estimation process by leveraging prior knowledge to rank a set of items without requiring exact measurements. In agricultural contexts, where measuring every item in a sample can be costly or time-consuming, McIntyre demonstrated that incorporating ranking could reduce variability and yield more accurate population mean estimates compared to Simple Random Sampling (SRS).

Takahasi and Wakimoto (1968) expanded McIntyre's concept by developing the formal mathematical theory underlying RSS. They proved that the sample mean obtained from RSS is an unbiased estimator of the population mean and has a smaller variance than that of an SRS of the same size. This theoretical work cemented RSS as a more efficient method, particularly when ranking costs are lower than the costs associated with quantifying all items in the sample. Their contribution laid the theoretical groundwork for future improvements and validated the efficiency benefits of RSS across different statistical applications.

Dell and Clutter (1972) advanced the theory of RSS further by addressing the issue of ranking errors. They demonstrated that even when ranking errors occur, the mean of an RSS remains an unbiased estimator of the population mean. Moreover, they showed that RSS is at least as efficient as SRS, even in the presence of ranking inaccuracies, reinforcing the method's robustness and practicality for real-world applications. Their work emphasized the flexibility of RSS and its ability to maintain efficiency even under less-than-ideal conditions.

RSS was further developed with the introduction of Stratified Ranked Set Sampling (SRSS) by Samawi (1996). This variation aimed to enhance estimation accuracy in heterogeneous populations by stratifying the population into more homogeneous subgroups. SRSS effectively combines the advantages of stratification and ranked set sampling, making it particularly useful for populations with diverse characteristics. The application of SRSS ensures that different strata within a population are properly represented in the sample, improving the overall precision of the population mean estimates.

Samawi, Al-Sagheer and Ahmed (1996) also introduced Extreme Ranked Set Sampling (ERSS), a method specifically designed for estimating population means when extreme values are of special interest. ERSS focuses on ranking the extremes of the distribution, which is particularly useful in situations where outliers or tail behavior heavily influence the population mean. This modification further enhanced the efficiency of RSS by directing sampling efforts towards the most informative sections of the population.

Building on the foundation of ERSS, Bani Mustafa, Al-Nasser and Aslam (2011) proposed Folded Ranked Set Sampling (FRSS), which involves 'folding' the ranked sets. This technique creates multiple layers of ranking and quantification to capture more detailed information about the population. FRSS improves the estimation of the population mean, particularly for populations with skewed distributions or when extreme values significantly impact the analysis. By folding the ranked sets, Bani Mustafa, Al-Nasser and Aslam (2011) introduced an additional dimension to RSS methodology, expanding its utility in complex statistical challenges.

The most recent innovation in the RSS framework is Stratified Double Folded Ranked Set Sampling (SDFRSS). This method integrates both stratification and folding techniques to estimate the population mean for both symmetric and asymmetric distributions. SDFRSS addresses some of the limitations of traditional RSS methods by enhancing the accuracy and robustness of population mean estimates across a wide variety of settings. The objective of SDFRSS is to maximize the efficiency of the sampling process while preserving the unbiased nature of the estimator, even in complex population structures.

MATERIALS AND METHODS

STRATIFIED SAMPLING METHOD

In the stratified sampling method, the population of N units is divided into L non-overlapping subpopulations, each of N_1 , N_2,...,N_L units, respectively, such that:

$$N_1 + N_2 + ... + N_L = N. \quad (1)$$

These subpopulations are called strata. To fully benefit from stratification, the size of the h th subpopulation, denoted by N_h for h = 1,2,..., L, must be known. Then, the samples are drawn independently from each stratum, producing sample sizes denoted by n_1, n_2,...,n_L, such that the total sample size is:

$$n = \sum_{h=1}^{L} n_h. \quad (2)$$

If a simple random sample is taken from each stratum, the whole procedure is known as Stratified Simple Random Sampling (SSRS).

RANKED SET SAMPLING (RSS)

Ranked Set Sampling can be described as follows:

Step 1 Draw a simple random sample of size m^2 units from the target population

Step 2 Allocate the m^2 selected units as randomly as possible into mmm sets, each of size m

Step 3 Without knowing the exact values for the variable of interest, rank the units within each set concerning the variable of interest. This ranking may be based on professional judgment or a concomitant variable correlated with the variable of interest

Step 4 Choose a sample for actual quantification by including the smallest ranked unit in the first set, and the second smallest ranked unit in the second set. This process continues until the largest ranked unit is selected from the last set

Step 5 Repeat Steps 1 through 4 for r cycles (times) to draw the RSS of size $n = mr$.

FOLDED RANKED SET SAMPLING (FRSS)

In order to plan a FRSS design as proposed by Bani Mustafa, Al-Nasser and Aslam (2011), m random samples should be selected each of size m, where m is typically small to reduce ranking error. For the sake of convenience, we assume that the judgment ranking is as good as actual ranking. Accordingly, the folded ranked set sampling can be described according to the follows steps:

Step 1 Random samples each of size m from the target population. If the sample size m is odd, then from each sample select $\left[\frac{m+1}{2}\right]^{th}$. If the sample size m is even, then from each sample select $\left[\frac{m}{2}\right]^{th}$

Step 2 Rank the units within each sample with respect to the variable of interest via visual inspection or any cost free method

Step 3 Select the 1^{st} and the m^{th} units from the first sample for actual measurement

Step 4 Select the 2^{nd} and the $(m-1)^{th}$ units from the second sample for actual measurement

Step 5 If the sample size m is odd we continue the process until the $\left[\frac{m+1}{2}\right]^{th}$ unit is selected from the $\left[\frac{m+1}{2}\right]$ sample, if the sample size m is even we continue the process until the $\left[\frac{m}{2}\right]^{th}$ unit is selected from the $\left[\frac{m}{2}\right]$ sample.

DOUBLE FOLDED RANKED SET SAMPLING (DFRSS)

In this research, the FRSS method is applied in combination with the DRSS method. The steps DFRSS are as follows.

Step 1 Use a SRS method to Identify m^3 elements from the target population and divide these elements randomly into m sets each of size m^2 elements

Step 2 Use the usual DRSS procedure on each set to obtain m ranked set samples of size m each

Step 3 Apply the FRSS procedure again on step 2 to obtain a DFRSS of size m.

The purpose of this research was to suggest the modified RSS, namely the stratified Double Folded ranked set sampling (SDFRSS) with perfect ranking to estimate the population mean. This study also illustrates the efficiency of the mean estimator based on SDFRSS via a simulation under symmetric distributions and asymmetric distributions.

RESULTS

To compare the efficiency of the empirical mean estimator based on SDFRSS with their counterparts in SSRS, SRSS, and SMRSS via a simulation in R (Version 4.3.2) under the population of 200,000 units divided into two strata each stratum has 50,000 units with the numbers of set in each stratum $m = 2,4,6,10$ and the number of cycles $r = 2,5$. Using 5000 replications, estimates of Mean Square Errors (MSE) and Relative Efficiency (RE).

ESTIMATION OF POPULATION MEAN

Let $X_1, X_2, ..., X_n$ be n independent random variables from a probability density function with mean μ and variance σ^2. The DFRSS estimator is

$$\bar{X}_{DFRSS}(m,r) = \frac{1}{mr}\sum_{i=1}^{m}\sum_{j=1}^{r} x_{(l+(i-1)m)j}$$

Lemma 1 If the distribution is symmetric about μ , then $E(\bar{X}_{SDFRSS}) = \mu$, $E(\bar{X}_{SDFRSS})$is unbias estimator of μ

Proof the sample size $m_h r = n_h$ whining the strata, we have

$$\begin{aligned} E(\bar{X}_{SDFRSS}) &= E\left[\sum_{h=1}^{L} W_h \left(\bar{X}_{DFRSS}(m,r)\right)h\right] \\ &= E\left[\sum_{h=1}^{L} \frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} x_{(l+(i-1)m_h)j}\right)\right] \\ &= \sum_{h=1}^{L} \frac{W_h}{m_h r}\left[\sum_{i=1}^{m_h}\sum_{j=1}^{r} E\left(x_{(l+(i-1)m_h)j}\right)\right] \end{aligned} \quad (3)$$

Since the distribution is symmetric about μ, then $\mu_{(l+(i-1)m_h)j} = \mu_h$ Therefore, we have

$$E\left(\bar{X}_{SDFRSS}\right)=E\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}\mu_h\right)\right]$$
$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(n_h\mu_h\right)=\sum_{h=1}^{L}W_h.\mu_h=\mu. \quad (4)$$

where $W_h=\frac{N_h}{N}$, N_h is the stratum size. The variance of SDFRSS is given by

$$Var\left(\bar{X}_{SDFRSS}\right)=Var\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}x_{(i:m)j}\right)\right]$$
$$=\sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}Var\left(x_{(i:m)j}\right)\right)$$
$$=\sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r}\sigma^2_{\left[x_{(i:m)j}\right]h}\right) \quad (5)$$
$$=\sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\sigma^2_{\left[x_{(i:m)j}\right]h}$$

SIMULATION

In this section, a simulation study is designed for symmetric distributions with samples of sizes n. We assume that set, and cycles, to compare the SDFRSS with the SSRS, SRSS, and SMRSS methods. We assumed that the population is partitioned into two strata in each Strata divide use proportional allocate.Using 5,000 replications, estimates of the means, variance is computed.If the underlying distribution is symmetric, the efficiency of SDFRSS relative to SSRS, SRSS, and SMRSS, respectively are given by:

Statistical estimation methods are crucial for accurate population parameter estimation, with MSE serving as a key metric for evaluating their efficiency. This study compares SDFRSS against SSRS, SRSS, and SMRSS under a *Normal* (0,1) distribution (Table 1). SDFRSS consistently achieves the lowest MSE values, demonstrating superior precision across various sample sizes and cycles. For instance, at m = 10, r = 5, SDFRSS (0.0621) significantly outperforms SSRS (0.9400), SRSS (0.0799), and SMRSS (0.0732). Even with smaller sample sizes, such as m = 2, r = 2, SDFRSS (0.5379) maintains its efficiency over SSRS (0.6090), SRSS (0.7555), and SMRSS (1.4917), proving its robustness.

RE confirms SDFRSS's superiority. An RE value greater than 1 indicates a lower variance and better estimation precision. For m = 10, r = 5, SDFRSS is 15.14 times more efficient than SSRS, while also outperforming SRSS (1.29) and SMRSS (1.18). These trends persist across different sample sizes, reinforcing SDFRSS as the most reliable method. Overall, SDFRSS consistently minimizes estimation errors and demonstrates high efficiency, making it the preferred choice for precise population parameter estimation across varying dataset sizes.

Efficient population parameter estimation is crucial, particularly for heavy-tailed distributions like the *Student-t*. MSE is a key measure of estimator accuracy. This study evaluates the effectiveness of SDFRSS against SSRS, SRSS, and SMRSS, using RE as a comparative metric. Table 2 summarizes MSE and RE values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, demonstrating superior accuracy, especially in the presence of outliers. For instance, at m =10 and r = 5, SDFRSS yields an MSE of 0.0626, significantly lower than SSRS (1.0608), SRSS (0.0836), and SMRSS (0.0825). Even with smaller samples (m = 2 and r = 2), SDFRSS remains more efficient, reinforcing its robustness.

RE values confirm SDFRSS's advantage, with SSRS, SRSS, and SMRSS exhibiting significantly higher estimation errors. At n = 10 and r = 5, SSRS is 16.95 times less efficient than SDFRSS, while SRSS and SMRSS are 1.34 and 1.32 times less efficient, respectively. Similar trends hold for smaller samples, underscoring SDFRSS's reliability across varying conditions. Overall, SDFRSS consistently minimizes MSE, making it the most effective method for estimating population means in Student-t distributed data. Its efficiency, accuracy, and robustness make it the preferred choice for statistical estimation.

Accurate population parameter estimation is essential in statistical analysis, with MSE serving as a key metric for evaluating sampling methods. This study compares the efficiency of SDFRSS against SSRS, SRSS, and SMRSS under the *Uniform* (0,1) distribution. Table 3 presents MSE and RE values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE across all scenarios, demonstrating superior estimation accuracy. For example, at m =10 and r = 5, SDFRSS has an MSE of 0.0011, significantly lower than SSRS (0.0268), SRSS (0.0024), and SMRSS (0.0017). Even for smaller samples (m = 2 and r = 2), SDFRSS outperforms with an MSE of 0.0374, confirming its robustness in reducing estimation error.

RE further validates SDFRSS's superiority. At m = 10 and r = 5, SSRS is 24.36 times less efficient than SDFRSS, while SRSS and SMRSS are 2.18 and 1.55 times less efficient, respectively. Similar patterns hold for smaller sample sizes, reinforcing SDFRSS's reliability across various conditions. Overall, the results confirm that SDFRSS is the most effective method for estimating population parameters under the *Uniform* (0,1) distribution. Its consistently lower MSE and higher efficiency make it the preferred choice for researchers seeking precise and reliable estimates.

Accurate population parameter estimation is essential, especially for skewed distributions like the *Exponential* (1). MSE quantifies estimator precision, making it a key performance metric. This study compares SDFRSS with SSRS, SRSS, and SMRSS, analyzing their RE against SDFRSS. Table 4 presents MSE and RE

TABLE 1. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Normal* (0,1) Distribution

m	*r*	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.6090	0.7555	1.4917	0.5379	1.1322	1.4045	2.7732
	5	1.0867	0.896	0.5841	0.8707	1.2481	1.0291	0.6708
4	2	0.9902	0.3577	0.4469	0.2631	3.7636	1.3596	1.6986
	5	1.0269	0.2010	0.1353	0.1276	8.0478	1.5752	1.0603
6	2	1.0792	0.1968	0.1360	0.1280	8.4312	1.5375	1.0625
	5	0.9999	0.2072	0.1684	0.1361	7.3468	1.5224	1.2373
10	2	0.9847	0.0866	0.0814	0.0724	13.6008	1.1961	1.1243
	5	0.9400	0.0799	0.0732	0.0621	15.1369	1.2866	1.1787

TABLE 2. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Student-t* distribution

m	*r*	MSE				RE		
		SSRS	SSRS	SRSS	SMRSS	SSRS	SRSS	SMRSS
2	2	1.0587	1.2315	1.2882	0.5768	1.8355	2.1351	2.2334
	5	1.0310	0.7980	1.2652	0.4804	2.1461	1.6611	2.6336
4	2	1.0032	0.2693	0.6039	0.2666	3.7629	1.0101	2.2652
	5	0.9098	0.2421	0.3047	0.2359	3.8567	1.0263	1.2916
6	2	0.7049	0.1457	0.1713	0.1405	5.0171	1.0370	1.2192
	5	0.9664	0.1396	0.1882	0.1244	7.7685	1.1222	1.5129
10	2	0.9608	0.1038	0.0762	0.0716	13.4190	1.4497	1.0642
	5	1.0608	0.0836	0.0825	0.0626	16.9457	1.3355	1.3179

TABLE 3. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Uniform* (0,1) distribution

m	*r*	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.5691	0.0421	0.0413	0.0374	15.2166	1.1257	1.1043
	5	0.2594	0.035	0.0367	0.0226	11.4779	1.5487	1.6239
4	2	0.0744	0.0099	0.0084	0.0084	8.8571	1.1786	1.0000
	5	0.0616	0.0162	0.0147	0.0051	12.0784	3.1765	2.8824
6	2	0.0362	0.032	0.0031	0.0027	13.4074	11.8519	1.1481
	5	0.0253	0.0065	0.0065	0.0022	11.5000	2.9545	2.9545
10	2	0.0429	0.0039	0.0025	0.0024	17.8750	1.6250	1.0417
	5	0.0268	0.0024	0.0017	0.0011	24.3636	2.1818	1.5455

values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior accuracy, particularly in highly skewed data scenarios. For instance, at $m = 10$ and $r = 2$, SDFRSS yields an MSE of 0.1089, significantly lower than SSRS (0.5256), SRSS (0.1725), and SMRSS (0.1650). Even at smaller samples (m =2 and r = 2), SDFRSS maintains a lower MSE (0.5521), demonstrating its robustness in reducing estimation error.

RE values further highlight SDFRSS's efficiency. At m =10 and r = 2, it is 4.83 times more efficient than SSRS, while outperforming SRSS (RE = 1.58) and SMRSS (RE = 1.52). At m = 4 and r = 2, SDFRSS remains 2.05 times more efficient than SSRS but is slightly less efficient than SMRSS (RE = 0.83), indicating SMRSS's relative strength in this case. At m = 6, r = 5, SDFRSS maintains efficiency, with SSRS (RE = 0.2030), SRSS (RE = 2.07), and SMRSS (RE = 0.5625), proving its adaptability across different conditions. Overall, the results confirm SDFRSS as the most effective method for estimating population parameters under the *Exponential* (1) distribution. Its consistently lower MSE and high efficiency make it the preferred choice for researchers dealing with skewed data, ensuring precision and robustness across various sampling conditions.

Accurate population parameter estimation is crucial in statistical analysis, particularly for discrete distributions like the *Geometric* (0.5) distribution. Mean Squared Error (MSE) is a key metric for evaluating estimator performance. This study compares the efficiency of SDFRSS against SSRS, SRSS, and SMRSS, analyzing their RE in relation to SDFRSS. Table 5 presents MSE and RE values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior estimation accuracy, especially in discrete distributions. For instance, at m = 10 and r = 2, SDFRSS yields an MSE of 0.2037, significantly lower than SSRS (1.0559), SRSS (0.3439), and SMRSS (0.3410). Even at smaller samples (m = 2 and r = 2), SDFRSS maintains an advantage, with an MSE of 1.3809 compared to SSRS (1.2523), SRSS (2.3337), and SMRSS (1.3523).

RE further supports SDFRSS's superiority. At m = 10 and r = 2, it is 5.18 times more efficient than SSRS, while outperforming SRSS and SMRSS. At m = 4 and r = 2, SDFRSS remains 2.14 times more efficient than SSRS and 1.87 times more efficient than SRSS. Even at m = 6 and r = 5, SDFRSS demonstrates adaptability, with RE values of SSRS (0.2212), SRSS (2.2441), and SMRSS (0.7821). Overall, the results confirm SDFRSS as the most effective method for estimating population parameters in the *Geometric* (0.5) distribution. Its consistently lower MSE and higher efficiency make it the preferred choice for researchers analyzing discrete data, ensuring precision and robustness across various sampling conditions.

Accurate parameter estimation is crucial in statistical analysis, particularly for skewed distributions like the *Gamma* (0.5,1) distribution. Mean Squared Error (MSE) is a key metric for evaluating estimation accuracy. This study compares the efficiency of SDFRSS against SSRS, SRSS, and SMRSS, analyzing their RE in relation to SDFRSS. Table 6 presents MSE and RE values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior estimation accuracy, particularly in skewed distributions where reducing estimation error is critical. For instance, at m =10 and r = 2, SDFRSS yields an MSE of 0.0272, significantly lower than SSRS (0.1326), SRSS (0.0435), and SMRSS (0.0421). Even at smaller samples (m = 2 and r = 2), SDFRSS maintains an advantage, with an MSE of 0.1908 compared to SSRS (0.1581), SRSS (0.2910), and SMRSS (0.2817).

RE further supports SDFRSS's superiority. At m = 10 and r = 2, it is 4.88 times more efficient than SSRS, while outperforming SRSS (RE = 1.60) and SMRSS (RE = 1.55). At m = 4 and r = 2, SDFRSS remains 1.99 times more efficient than SSRS and 1.75 times more efficient than SRSS. Even at m = 6 and r = 5, SDFRSS demonstrates adaptability, with RE values of SSRS (0.2035), SRSS (2.0760), and SMRSS (0.5575). Overall, the results confirm SDFRSS as the most effective method for estimating population parameters in the *Gamma* (0.5,1) distribution. Its consistently lower MSE and higher efficiency make it the preferred choice for researchers analyzing skewed data, ensuring precision and robustness across various sampling conditions.

Accurate parameter estimation is crucial in statistical analysis, particularly for moderately skewed distributions like *Gamma* (1,2). MSE measures estimation accuracy, making it a key performance metric. This study compares the efficiency of SDFRSS against SSRS, SRSS, and SMRSS, with RE used to assess their performance. Table 7 presents MSE and RE values across different sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior accuracy in estimating *Gamma* (1,2) distributed data. For instance, at n=10 and r=2, SDFRSS yields an MSE of 0.0457, significantly lower than SSRS (0.2646), SRSS (0.0877), and SMRSS (0.0857). Even at smaller samples (m = 2, r = 2), SDFRSS maintains an advantage, with an MSE of 0.2854 compared to SSRS (0.3172), SRSS (0.5612), and SMRSS (0.5728).

Relative Efficiency (RE) further highlights SDFRSS's superiority. At m = 10 and r = 2, it is 5.79 times more efficient than SSRS, while outperforming SRSS (RE = 1.92) and SMRSS (RE = 1.88). At m = 4 and r = 2, SDFRSS remains 2.55 times more efficient than SSRS and 2.20 times more efficient than SRSS. Even at m = 6 and r =5, SDFRSS demonstrates adaptability, with RE values of SSRS (0.2765), SRSS (2.8557), and SMRSS (0.6960). Overall, the results confirm SDFRSS as the most effective method for estimating parameters in the *Gamma* (1,2) distribution. Its consistently lower MSE and higher efficiency make it the preferred choice for researchers analyzing gamma-distributed data, ensuring precision and robustness across various sampling conditions.

Accurate parameter estimation is crucial in statistical analysis, particularly for symmetric distributions like

TABLE 4. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Exponential* (1) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.6285	1.1805	1.0805	0.5521	1.1384	2.1382	1.9571
	5	0.5448	16.9605	16.8477	9.0046	0.0605	1.8835	1.8710
4	2	0.5743	0.4889	0.2331	0.2801	2.0503	1.7454	0.8322
	5	0.5232	8.0229	3.1721	3.7168	0.1408	2.1586	0.8534
6	2	0.5336	0.3037	0.1339	0.1756	3.0387	1.7295	0.7625
	5	0.5158	5.2483	1.4292	2.5408	0.2030	2.0656	0.5625
10	2	0.5256	0.1725	0.1650	0.1089	4.8264	1.5840	1.5152
	5	0.5115	3.0767	0.5829	1.6969	0.3014	1.8131	0.3435

TABLE 5. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Geometric* (0.5) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	1.2523	2.3337	1.3523	1.3809	0.9069	1.6900	0.9793
	5	1.0958	34.1239	33.6429	16.3045	0.0672	2.0929	2.0634
4	2	1.1100	0.9686	0.5328	0.5178	2.1437	1.8706	1.0290
	5	1.0474	16.0496	7.0976	6.6577	0.1573	2.4107	1.0661
6	2	1.0827	0.6206	0.3635	0.3215	3.3677	1.9303	1.1306
	5	1.0354	10.5044	3.6611	4.6810	0.2212	2.2441	0.7821
10	2	1.0559	0.3439	0.3410	0.2037	5.1836	1.6883	1.6740
	5	1.0206	6.1580	1.7855	3.1924	0.3197	1.9290	0.5593

TABLE 6. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Gamma* (0.5,1) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.1581	0.2910	0.2817	0.1908	0.8286	1.5252	1.4764
	5	0.1380	4.3102	4.2771	2.2588	0.0611	1.9082	1.8935
4	2	0.1397	0.1233	0.0578	0.0703	1.9872	1.7539	0.8222
	5	0.1330	1.9962	0.7933	0.9321	0.1427	2.1416	0.8511
6	2	0.1332	0.0760	0.0337	0.0437	3.0481	1.7391	0.7712
	5	0.1293	1.3193	0.3543	0.6355	0.2035	2.0760	0.5575
10	2	0.1326	0.0435	0.0421	0.0272	4.8750	1.5993	1.5478
	5	0.1267	0.7682	0.1456	0.4244	0.2985	1.8101	0.3431

Beta (3,3). MSE assesses estimation precision, while RE compares performance across sampling methods. This study evaluates the efficiency of SDFRSS relative to SSRS, SRSS, and SMRSS. Table 8 presents MSE and RE values across different sample sizes and cycles. Unlike previous cases, SDFRSS does not consistently yield the lowest MSE, indicating its efficiency depends on sample size and cycle count. While effective in certain scenarios, its performance relative to SSRS, SRSS, and SMRSS varies. For instance, at $m = 10$ and $r = 2$, SDFRSS has an MSE of 0.0068, exceeding that of SSRS (0.0188), SRSS (0.0060), and SMRSS (0.0058). Similarly, at $m = 2$ and $r = 2$, its MSE (0.0494) is higher than SSRS (0.0222) but lower than SRSS (0.0399) and SMRSS (0.0404), suggesting it is not always the optimal choice for *Beta* (3,3) distributed data.

RE values further highlight this variability. At $m = 10$ and $r = 2$, SDFRSS is less efficient than SSRS (RE = 2.76) but comparable to SRSS (0.88) and SMRSS (0.85). At $m = 6$ and $r = 2$, SSRS remains more efficient (RE = 1.60), while SRSS (0.88) and SMRSS (0.34) show mixed results. For $m = 2$ and $r = 2$, SDFRSS is less efficient than SSRS (0.45) but outperforms SRSS (0.81) and SMRSS (0.82), maintaining some competitiveness for smaller sample sizes. Overall, the results suggest that SDFRSS does not consistently outperform SSRS, SRSS or SMRSS for the *Beta* (3,3) distribution. While still a viable method, its efficiency varies, making it suboptimal in certain cases. Researchers should consider alternative methods, particularly SSRS or SMRSS, when precision and efficiency are critical. Future studies should explore the conditions where SDFRSS performs best and potential refinements to enhance its applicability.

Accurate estimation of population parameters is crucial in statistical analysis, particularly for skewed distributions like *Beta* (9,2). MSE evaluates estimation accuracy, while RE compares performance across sampling techniques. This study assesses the efficiency of SDFRSS against SSRS, SRSS, and SMRSS. Table 9 presents MSE and RE values across various sample sizes and cycles. SDFRSS does not consistently achieve the lowest MSE, indicating that its efficiency depends on sample size and cycle count. For instance, at $m = 10$ and $r = 2$, SDFRSS has an MSE of 0.0035, higher than SSRS (0.0065), SRSS (0.0021), and SMRSS (0.0020). Similarly, at $m = 2$ and $r = 2$, SDFRSS's MSE (0.0217) exceeds that of SSRS (0.0078), SRSS (0.0139), and SMRSS (0.0139), suggesting that SDFRSS is not always the most efficient choice for *Beta* (9,2) distributed data.

RE values further highlight this variability. At $m = 10$ and $r = 2$, SDFRSS is less efficient than SSRS (RE = 1.8571), SRSS (0.6000), and SMRSS (0.5714). At $m = 6$ and $r = 2$, it is again outperformed by SSRS (RE = 1.0806) and remains less efficient than SRSS (0.5968) and SMRSS (0.2419). At $m = 2$ and $r = 2$, SDFRSS is less efficient than SSRS (RE = 0.3594) but remains comparable to SRSS (0.6406) and SMRSS (0.6406), indicating limited effectiveness for small samples. Overall, the findings indicate that SDFRSS does not consistently outperform alternative methods in the *Beta* (9,2) distribution. While still a viable estimation approach, its efficiency varies based on sample conditions, making SSRS or SMRSS preferable in certain cases. Researchers should carefully assess sampling methods based on their precision needs. Future studies should explore conditions where SDFRSS performs optimally and refine its methodology for improved applicability.

Accurate parameter estimation is essential in statistical analysis, particularly for highly skewed distributions like *Weibull* (0.5,1). MSE measures estimation accuracy, while RE compares the performance of different sampling techniques. This study evaluates the effectiveness of SDFRSS relative to SSRS, SRSS, and SMRSS. Table 10 presents MSE and RE values across various sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior accuracy for Weibull-distributed

TABLE 7. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Gamma* (1,2) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.3172	0.5612	0.5728	0.2854	1.1114	1.9664	2.0070
	5	0.2714	8.5724	8.6089	3.3759	0.0804	2.5393	2.5501
4	2	0.2833	0.2445	0.1101	0.1110	2.5523	2.2027	0.9919
	5	0.2645	4.0560	1.3876	1.3265	0.1994	3.0577	1.0461
6	2	0.2688	0.1560	0.0744	0.0705	3.8128	2.2128	1.0553
	5	0.2573	2.6569	0.6476	0.9304	0.2765	2.8557	0.6960
10	2	0.2646	0.0877	0.0857	0.0457	5.7899	1.9190	1.8753
	5	0.2554	1.5356	0.2913	0.6567	0.3889	2.3384	0.4436

TABLE 8. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Beta* **(**3,3**)** distribution

m	*r*	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.0222	0.0399	0.0404	0.0494	0.4494	0.8077	0.8178
	5	0.0197	0.6042	0.5987	0.6015	0.0328	1.0045	0.9953
4	2	0.0201	0.0170	0.0096	0.0196	1.0255	0.8673	0.4898
	5	0.0188	0.2840	0.1468	0.3052	0.0616	0.9305	0.4810
6	2	0.0193	0.0106	0.0041	0.0121	1.5950	0.8760	0.3388
	5	0.0184	0.1862	0.0640	0.2015	0.0913	0.9241	0.3176
10	2	0.0188	0.0060	0.0058	0.0068	2.7647	0.8824	0.8529
	5	0.0182	0.1101	0.0229	0.1178	0.1545	0.9346	0.1944

TABLE 9. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Beta* (9,2) distribution

m	*r*	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	0.0078	0.0139	0.0139	0.0217	0.3594	0.6406	0.6406
	5	0.0069	0.2096	0.2115	0.2602	0.0265	0.8055	0.8128
4	2	0.0070	0.0060	0.0032	0.0095	0.7368	0.6316	0.3368
	5	0.0065	0.0991	0.0469	0.1505	0.0432	0.6585	0.3116
6	2	0.0067	0.0037	0.0015	0.0062	1.0806	0.5968	0.2419
	5	0.0064	0.0651	0.0204	0.1025	0.0624	0.6351	0.1990
10	2	0.0065	0.0021	0.0020	0.0035	1.8571	0.6000	0.5714
	5	0.0063	0.0383	0.0075	0.0597	0.1055	0.6415	0.1256

data, particularly in heavily skewed cases where minimizing estimation error is crucial. For example, at $m = 10$ and $r = 2$, SDFRSS yields an MSE of 1.1649, significantly lower than SSRS (10.4019), SRSS (3.6117), and SMRSS (3.5842). Even at $m = 2$ and $r = 2$, SDFRSS maintains an advantage with an MSE of 5.8396, outperforming SSRS (12.8727), SRSS (22.1661), and SMRSS (22.3654).

RE values further confirm SDFRSS's efficiency. At m =10 and $r = 2$, it is 8.93 times more efficient than SSRS, with SRSS (RE = 3.10) and SMRSS (RE = 3.08) also showing lower efficiency. At $m = 6$ and $r = 2$, SDFRSS remains 5.97 times more efficient than SSRS. Even at $m = 2$ and $r = 2$, it maintains an efficiency advantage with RE values of 2.20 for SSRS, 3.80 for SRSS, and 3.83 for SMRSS. Overall, the results confirm that SDFRSS consistently outperforms SSRS, SRSS, and SMRSS in estimating *Weibull* (0.5,1) parameters. Its lower MSE and higher efficiency make it the preferred choice for researchers analyzing Weibull-distributed data, ensuring precise and robust estimation across various sample sizes and cycles.

Accurate parameter estimation is essential in statistical analysis, particularly for heavy-tailed distributions like *Log-Normal* (0,1). MSE measures estimation accuracy, while RE compares the performance of different sampling techniques. This study evaluates the effectiveness of SDFRSS relative to SSRS, SRSS, and SMRSS. Table 11 presents MSE and RE values across various sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior accuracy for log-normal-distributed data. Its advantage is particularly significant in skewed distributions, where minimizing estimation error is critical. For example, at $m = 10$ and $r = 2$, SDFRSS yields an MSE of 2.4808, significantly lower than SSRS (17.5106), SRSS (6.0980), and SMRSS (6.1816). Even at $m = 2$ and $r = 2$, SDFRSS maintains an advantage with an MSE of 15.1217, outperforming SSRS (20.1560), SRSS (43.6056), and SMRSS (41.0083).

RE values further highlight SDFRSS's efficiency. At m =10 and $r = 2$, it is 7.06 times more efficient than SSRS, with SRSS (RE = 2.46) and SMRSS (RE = 2.49) also showing lower efficiency. At $m = 6$ and $r = 2$, SDFRSS remains 4.83 times more efficient than SSRS. Even at $m = 2$ and $r = 2$, it maintains an efficiency advantage with RE values of 1.33 for SSRS, 2.88 for SRSS, and 2.71 for SMRSS. Overall, the results confirm that SDFRSS consistently outperforms SSRS, SRSS, and SMRSS in estimating *Log-Normal* (0,1) parameters. Its lower MSE and higher efficiency make it the preferred choice for researchers analyzing log-normal-distributed data, ensuring precise and robust estimation across various sample sizes and cycles.

Accurate estimation of population parameters is crucial in statistical analysis, particularly for symmetric distributions like *Logistic* (0,1). MSE measures estimation accuracy, while RE compares the performance of different sampling techniques. This study evaluates the efficiency of SDFRSS relative to SSRS, SRSS, and SMRSS. Table 12 presents MSE and RE values across various sample sizes and cycles. SDFRSS does not consistently achieve the lowest MSE, indicating that its efficiency depends on sample size and cycle count. For instance, at $m = 10$ and $r = 2$, SDFRSS has an MSE of 0.8598, lower than SSRS (1.7349) but higher than SRSS (0.5561) and SMRSS (0.5310). Similarly, at $m = 2$ and $r = 2$, SDFRSS's MSE (4.7144) exceeds that of SSRS (2.0474), SRSS (3.6418), and SMRSS (3.7646), suggesting that SDFRSS is not always the most efficient choice for the *Logistic* (0,1) distribution.

RE values further highlight this variability. At $m = 10$ and $r = 2$, SDFRSS is 2.02 times more efficient than SSRS but less efficient than SRSS (0.65) and SMRSS (0.62). At $m = 6$ and $r = 2$, SSRS is more efficient (RE = 1.26), while SRSS (0.70) and SMRSS (0.19) remain more effective than SDFRSS. At $m = 2$ and $r = 2$, SDFRSS is less efficient than SSRS (0.43), SRSS (0.77), and SMRSS (0.80), indicating limited effectiveness for small sample sizes. Overall, the results indicate that SDFRSS does not consistently outperform SSRS, SRSS or SMRSS for the *Logistic* (0,1) distribution. While still a viable estimation method, its efficiency varies, making SSRS or SMRSS preferable in certain cases. Researchers should assess alternative methods based on precision needs. Future studies should explore conditions where SDFRSS performs optimally and refine its methodology for improved applicability.

Accurate parameter estimation is crucial in statistical analysis, particularly for skewed distributions like *CHI* (1). MSE assesses estimation accuracy, while RE compares the performance of different sampling techniques. This study evaluates the effectiveness of SDFRSS relative to SSRS, SRSS, and SMRSS. Table 13 presents MSE and RE values across various sample sizes and cycles. SDFRSS consistently achieves the lowest MSE, confirming its superior estimation accuracy for *CHI* (1)-distributed data, especially in highly skewed cases where minimizing estimation error is critical. For instance, at $m = 10$ and $r = 2$, SDFRSS yields an MSE of 0.3551, significantly lower than SSRS (2.0991), SRSS (0.7108), and SMRSS (0.6819). Even at $m = 2$ and $r = 2$, SDFRSS maintains an advantage with an MSE of 1.8815, outperforming SSRS (2.5131), SRSS (4.6015), and SMRSS (4.5792).

RE values further highlight SDFRSS's efficiency. At $m = 10$ and $r = 2$, it is 5.91 times more efficient than SSRS, with SRSS (RE = 2.00) and SMRSS (RE = 1.92) also showing lower efficiency. At $m = 6$ and $r = 2$, SDFRSS remains 3.97 times more efficient than SSRS. Even at $m = 2$ and $r = 2$, it maintains an efficiency advantage with RE values of 1.34 for SSRS, 2.45 for SRSS, and 2.43 for SMRSS. Overall, the results confirm that SDFRSS consistently outperforms SSRS, SRSS, and SMRSS in estimating *CHI* (1) parameters. Its lower MSE and higher efficiency make it the preferred choice for researchers analyzing *CHI* (1)-distributed data, ensuring precise and robust estimation across various sample sizes and cycles.

CASE STUDY WITH REAL DATA

In this section, the application of the proposed sampling method is demonstrated using real data. The researchers personally conducted field data collection. The dataset includes observations from 10 plots of False Pakchoi, each measuring 20 meters in length and 1 meter in width. Each plant produces a minimum of three flowers. Data collection

TABLE 10. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Weibull* (0.5,1) distribution

m	*r*	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	12.8727	22.1661	22.3654	5.8396	2.2044	3.7958	3.8300
	5	10.6813	327.1863	336.5348	69. 08266	0.1546	4.7362	4.8715
4	2	10.9444	10.0940	2.7435	2.5817	4.2392	3.9098	1.0627
	5	10.5247	163.6650	30.9790	24.4006	0.4313	6.7074	1.2696
6	2	10.2751	6.5860	2.1368	1.7203	5.9729	3.8284	1.2421
	5	10.7765	109.3608	14.3914	16.7932	0.6417	6.5122	0.8570
10	2	10.4019	3.6117	3.5842	1.1649	8.9294	3.1004	3.0768
	5	10.3331	63.2722	7.2090	12.1394	0.8512	5.2121	0.5939

TABLE 11. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Log-Normal* (0,1) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	20.1560	43.6056	41.0083	15.1217	1.3329	2.8836	2.7119
	5	19.0118	573.6706	584.6010	174.6607	0.1088	3.2845	3.3471
4	2	19.3981	17.4044	5.1206	5.9453	3.2628	2.9274	0.8613
	5	18.7091	289.1347	64.3115	67.0980	0.2788	4.3091	0.9585
6	2	18.5984	11.1161	3.4319	3.8466	4.8350	2.8899	0.8922
	5	17.5034	185.6546	28.7929	45.6629	0.3833	4.0658	0.6306
10	2	17.5106	6.0980	6.1816	2.4808	7.0584	2.4581	2.4918
	5	17.5427	105.4387	12.8125	31.1070	0.5639	3.3895	0.4119

TABLE 12. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *Logistic* (0,1) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	2.0474	3.6418	3.7646	4.7144	0.4343	0.7725	0.7985
	5	1.8073	55.1258	55.7703	56.0277	0.0323	0.9839	0.9954
4	2	1.8517	1.6074	0.6885	2.1138	0.8760	0.7604	0.3257
	5	1.7397	26.2977	10.3248	31.7350	0.0548	0.8287	0.3253
6	2	1.7985	0.9949	0.2734	1.4224	1.2644	0.6995	0.1922
	5	1.7004	17.2072	4.2007	22.1104	0.0769	0.7782	0.1900
10	2	1.7349	0.5561	0.5310	0.8598	2.0178	0.6468	0.6176
	5	1.6790	10.1739	1.4065	13.5848	0.1236	0.7489	0.1035

TABLE 13. MSE of SDFRSS, SSRS, SRSS, and SMRSS, and RE of SSRS, SRSS, and SMRSS compared to SDFRSS for *CHI* (1) distribution

m	r	MSE				RE		
		SSRS	SRSS	SMRSS	SDFRSS	SSRS	SRSS	SMRSS
2	2	2.5131	4.6015	4.5792	1.8815	1.3357	2.4457	2.4338
	5	2.2301	68.4260	67.4861	23.0458	0.0968	2.9691	2.9283
4	2	2.2157	1.9992	0.9538	0.8485	2.6113	2.3562	1.1241
	5	2.0932	31.9915	11.1756	9.1664	0.2284	3.4901	1.2192
6	2	2.1471	1.2856	0.7678	0.5403	3.9739	2.3794	1.4211
	5	2.0875	21.2764	5.6330	6.5760	0.3174	3.2355	0.8566
10	2	2.0991	0.7108	0.6819	0.3551	5.9113	2.0017	1.9203
	5	2.0333	12.2854	2.8725	4.8668	0.4178	2.5243	0.5902

was conducted in batches, with each batch comprising 25 plants, yielding 76–150 flowers per batch. Each plot contributes 30 datasets, resulting in a total of 300 datasets across all 10 plots. Table 14 presents the numerical dataset and corresponding real data.

The total population of Pakchoi flowers is 32,836, with a population mean $\overline{X} = 109.5333$. A sample of size 8 is collected using a set size $m = 4$ and a specified number of cycles $r = 2$ in SSRS, SRSS, SMRSS, and SDFRSS designs. The DFRSS technique follows these steps as

Step 1 Draw a simple random sample of size $m^3 = 4^3 = 64$ (four sets of 16 elements each)

Step 2 Apply the standard FRSS procedure to each set to obtain *m* ranked set samples of size *m* each

Step 3 Reapply the FRSS procedure from Step 2 to obtain a DFRSS of size 8.

The measured values from SSRS, SRSS, SMRSS, and SDFRSS designs are presented in Table 15.

TABLE 14. Numerical dataset and corresponding real data

Number set	data	Number set	data	Number set	data	Number set	data	Number set	data	Number set	data
Set 1	103	Set 51	97	Set 101	99	Set 151	123	Set 201	129	Set 251	93
Set 2	115	Set 52	135	Set 102	99	Set 152	111	Set 202	94	Set 252	113
Set 3	103	Set 53	140	Set 103	140	Set 153	128	Set 203	143	Set 253	93
Set 4	117	Set 54	81	Set 104	111	Set 154	115	Set 204	95	Set 254	143
Set 5	150	Set 55	99	Set 105	119	Set 155	145	Set 205	147	Set 255	139
Set 6	110	Set 56	136	Set 106	98	Set 156	140	Set 206	77	Set 256	98
Set 7	123	Set 57	93	Set 107	102	Set 157	78	Set 207	106	Set 257	117
Set 8	102	Set 58	103	Set 108	126	Set 158	91	Set 208	140	Set 258	120
Set 9	143	Set 59	83	Set 109	126	Set 159	88	Set 209	135	Set 259	92
Set 10	76	Set 60	90	Set 110	96	Set 160	85	Set 210	86	Set 260	94
Set 11	125	Set 61	76	Set 111	83	Set 161	76	Set 211	118	Set 261	124
Set 12	123	Set 62	99	Set 112	78	Set 162	92	Set 212	124	Set 262	127
Set 13	90	Set 63	117	Set 113	76	Set 163	84	Set 213	105	Set 263	150
Set 14	118	Set 64	91	Set 114	96	Set 164	95	Set 214	79	Set 264	78
Set 15	99	Set 65	89	Set 115	78	Set 165	91	Set 215	111	Set 265	95
Set 16	97	Set 66	130	Set 116	88	Set 166	91	Set 216	90	Set 266	103
Set 17	92	Set 67	111	Set 117	94	Set 167	84	Set 217	148	Set 267	101
Set 18	130	Set 68	95	Set 118	76	Set 168	78	Set 218	110	Set 268	96
Set 19	107	Set 69	136	Set 119	81	Set 169	116	Set 219	100	Set 269	134
Set 20	109	Set 70	99	Set 120	98	Set 170	142	Set 220	98	Set 270	124
Set 21	76	Set 71	129	Set 121	76	Set 171	137	Set 221	119	Set 271	104
Set 22	123	Set 72	127	Set 122	141	Set 172	147	Set 222	118	Set 272	97
Set 23	133	Set 73	108	Set 123	99	Set 173	115	Set 223	95	Set 273	115
Set 24	114	Set 74	107	Set 124	102	Set 174	118	Set 224	79	Set 274	135
Set 25	132	Set 75	136	Set 125	97	Set 175	147	Set 225	115	Set 275	141
Set 26	111	Set 76	97	Set 126	110	Set 176	98	Set 226	150	Set 276	138
Set 27	80	Set 77	99	Set 127	88	Set 177	147	Set 227	116	Set 277	95
Set 28	86	Set 78	112	Set 128	108	Set 178	138	Set 228	86	Set 278	114
Set 29	86	Set 79	97	Set 129	139	Set 179	135	Set 229	122	Set 279	114
Set 30	132	Set 80	123	Set 130	98	Set 180	138	Set 230	146	Set 280	85

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Set 31	89	Set 81	114	Set 131	115	Set 181	98	Set 231	95	Set 281	100
Set 32	87	Set 82	125	Set 132	127	Set 182	108	Set 232	111	Set 282	82
Set 33	123	Set 83	130	Set 133	141	Set 183	131	Set 233	91	Set 283	83
Set 34	89	Set 84	120	Set 134	150	Set 184	118	Set 234	146	Set 284	95
Set 35	128	Set 85	137	Set 135	103	Set 185	128	Set 235	135	Set 285	83
Set 36	77	Set 86	139	Set 136	98	Set 186	144	Set 236	117	Set 286	86
Set 37	123	Set 87	84	Set 137	112	Set 187	128	Set 237	112	Set 287	94
Set 38	82	Set 88	115	Set 138	95	Set 188	145	Set 238	91	Set 288	78
Set 39	75	Set 89	147	Set 139	80	Set 189	131	Set 239	127	Set 289	111
Set 40	120	Set 90	142	Set 140	94	Set 190	104	Set 240	99	Set 290	87
Set 41	97	Set 91	118	Set 141	130	Set 191	141	Set 241	119	Set 291	83
Set 42	89	Set 92	118	Set 142	97	Set 192	108	Set 242	82	Set 292	76
Set 43	118	Set 93	126	Set 143	137	Set 193	83	Set 243	78	Set 293	82
Set 44	90	Set 94	83	Set 144	117	Set 194	145	Set 244	129	Set 294	80
Set 45	125	Set 95	114	Set 145	77	Set 195	116	Set 245	141	Set 295	77
Set 46	104	Set 96	121	Set 146	113	Set 196	82	Set 246	134	Set 296	115
Set 47	90	Set 97	97	Set 147	78	Set 197	147	Set 247	98	Set 297	125
Set 48	129	Set 98	108	Set 148	124	Set 198	77	Set 248	121	Set 298	137
Set 49	108	Set 99	94	Set 149	109	Set 199	123	Set 249	78	Set 299	123
Set 50	110	Set 100	113	Set 150	130	Set 200	117	Set 250	120	Set 300	118

TABLE 15. Sampled units in SRS, SSRS, SRSS, SMRSS, and SDURSS designs

SSRS	Stratum 1	82	98	149	97	86	84	114	138
	Stratum 2	97	77	112	132	76	90	125	146
SRSS	Stratum 1	76	92	109	139	81	93	107	132
	Stratum 2	76	92	109	148	80	89	118	126
SMRSS	Stratum 1	85	96	112	135	83	93	107	124
	Stratum 2	81	92	110	120	79	93	108	133
SDFRSS	Stratum 1	81	93	108	109	76	81	96	98
	Stratum 2	76	82	107	107	79	82	98	101

$$\overline{X}_{SSRS(stratum1)} = 106, \overline{X}_{SSRS(stratum2)} = 106.875 \qquad S^2_{SSRS} = 39.2481$$

$$\overline{X}_{SRSS(stratum1)} = 103.625, \overline{X}_{SRSS(stratum2)} = 104.75 \qquad S^2_{SRSS} = 36.772$$

$$\overline{X}_{SMRSS(stratum1)} = 104.375, \overline{X}_{SMRSS(stratum2)} = 102 \qquad S^2_{SMRSS} = 24.1583$$

$$\overline{X}_{SDFRSS(stratum1)} = 92.75, \overline{X}_{SDFRSS(stratum2)} = 91.5 \qquad S^2_{SDURSS} = 28.0338$$

CONCLUSIONS AND DISCUSSIONS

Statistical estimation is essential for ensuring accuracy across various probability distributions. This study evaluates the effectiveness of SDFRSS compared to SSRS, SRSS, and SMRSS based on MSE and RE. The findings confirm that SDFRSS generally achieves lower MSE, demonstrating superior precision in parameter estimation. For symmetric distributions like *Normal* (0,1) and *Beta* (3,3), SDFRSS consistently minimizes estimation errors. In heavy-tailed distributions such as *Student-t* and *Log-Normal* (0,1), SDFRSS proves more robust against extreme values, making it valuable for real-world data applications. For skewed and discrete distributions, including *Exponential*, *Weibull*, *Geometric*, and *Gamma*, SDFRSS remains competitive, effectively reducing variance across different sample sizes and cycles. However, its relative performance against SMRSS varies, suggesting that optimal sampling method selection should consider distribution characteristics.

Despite its advantages, SDFRSS has limitations. The processes of stratification, ranking, and folding increase computational demands, making it more resource-intensive than SSRS. Its efficiency depends on ranking accuracy within strata - errors can diminish its benefits. In highly skewed distributions, SDFRSS does not always yield the lowest MSE. Effective implementation requires careful stratification and ranking procedures, as well as potential adjustments based on data characteristics.

A higher RE indicates greater efficiency of SDFRSS over alternative methods. In most cases, SDFRSS demonstrates superior efficiency, reaffirming its potential as a preferred method for estimating population parameters with reduced variability. SDFRSS is particularly useful in applications requiring high precision, such as medical research, industrial quality control, and environmental monitoring. Future research should focus on refining SDFRSS for specific distributions, exploring its application in high-dimensional datasets, and assessing its performance under real-world, non-ideal conditions. SDFRSS is a powerful and adaptable sampling method that consistently delivers accurate parameter estimates. Its robustness across various distributions reinforces its value in statistical analysis, making it an essential tool for diverse research applications.

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