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Monitoring Stability of Nonconformities Per Unit with Efficient Memory-Type Structure

(Memantau Kestabilan Ketakakuran Setiap Unit dengan Struktur Jenis Memori yang Cekap)

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ABSTRACT

For monitoring the qualitative characteristics of interest attribute control charts are recommended. Poisson control charts are most frequently used to track the number of nonconformities per unit in industrial processes during the inspection. This study introduces a Poisson EWMA (PEWMA) charting scheme using a progressive paradigm (PEWMA-p) to improve the sensitivity of the PEWMA chart for possible combinations of smoothing parameters. The proposed PEWMA-p chart has been developed for Poisson processes to monitor nonconformities per unit. The run-length (RL) profiles of the proposed PEWMA-p chart have been computed using extensively applied Monte Carlo simulations. The proposed PEWMA-p chart super-passes the counterparts for monitoring small shifts. An illustrative implementation related to the food quality for the proposed PEWMA-p with existing competitors is also part of the study which highlights the importance of the proposed chart.

Keywords: Attribute characteristics; control chart; EWMA chart; memory-type charting schemes; Poisson distribution

ABSTRAK

Untuk memantau ciri kualitatif minat, carta kawalan atribut disyorkan. Carta kawalan Poisson paling kerap digunakan untuk menjejaki bilangan ketidakpatuhan per unit dalam proses perindustrian semasa pemeriksaan. Penyelidikan ini memperkenalkan skema carta Poisson EWMA (PEWMA) menggunakan paradigma progresif (PEWMA-p) untuk meningkatkan sensitiviti carta PEWMA untuk kemungkinan gabungan parameter pelicin. Carta PEWMA-p yang dicadangkan telah dibangunkan untuk proses Poisson untuk memantau ketidakpatuhan setiap unit. Profil panjang larian (RL) carta PEWMA-p yang dicadangkan telah dikira menggunakan simulasi Monte Carlo yang digunakan secara meluas. Carta PEWMA-p yang dicadangkan melepasi rakan sejawat untuk memantau anjakan kecil. Pelaksanaan ilustrasi berkaitan kualiti makanan untuk PEWMA-p yang dicadangkan dengan pesaing sedia ada juga merupakan sebahagian daripada kajian yang menyerlahkan kepentingan carta yang dicadangkan.

Kata kunci: Atribut ciri; carta EWMA; carta kawalan; skim carta jenis ingatan; taburan Poisson

INTRODUCTION

In industrial and service environments, irregularities are just one of the many reasons why process parameters can change while they are in use. These variations, sometimes referred to as specific causes of variation, may affect the process's effectiveness and the quality of the product. The difficulty that practitioners frequently have is not being able to predict just how much these parameter changes will be. As a result, they usually provide a threshold a fixed shift size, that they deem important for their particular procedure (Alevizakos & Koukouvinos 2020; cf. Montgomery 2019). An important instrument for statistical quality control is the control chart (Zhou, Shu & Jiang 2016). After that, the control chart they select is made to quickly recognize changes that are larger than or equal to this predetermined shift size (Ghasemian & Noorossana 2024). But if a different shift size happens in the process, the chart might not notice it the right way, which could cause the response time to be delayed. They are used to keep an eye on important process metrics such as the variation, mean, and percentage of non-conforming products. Plotting against control limits, which are commonly denoted by an upper control limit (UCL) and a lower control limit (LCL), these charts employ a statistic generated by sample data. The process is said to be in control (IC) when the plotting statistic remains inside these control limits. On the other hand, the process is considered out-of-control (OOC) if the statistic intersects one of these control limits.

Discrete quality attributes are common in many real-world situations. These aspects entail classifying product units according to certain properties or features, as either non-conforming or conforming. Counting the number of flaws per item or unit is another interesting feature that makes it a pertinent quality attribute (Ahmad et al. 2024). In these kinds of scenarios, discrete probability distributions such as Poisson, Bernoulli, Binomial, or Geometric are better suited for modeling the data. In this study, we are primarily interested in the processes that follow Poisson distribution.

The Poisson EWMA (PEWMA) chart was proposed by Borror, Champ and Rigdon (1998) as a means of tracking failure data. Using the Markov chain method, they calculated the average run-length (ARL) profiles for their proposed PEWMA chart. The IC ARL and OOC ARL are customary denoted by ARL_0 and ARL_1 respectively. For the relative effectiveness of a chart its ARL_1 should be smaller than the competitor(s) at certain size of shift. Zhang et al. (2003) designed a Poisson double EWMA charting scheme for monitoring the mean of Poisson distribution. Testik, McCullough and Borrar (2006) investigated the impact of estimating error on the run-length profiles of the Poisson EWMA chart. Sheu and Chiu (2007) and Chen (2020) suggested the generally weighted moving average (GWMA) and double GWMA charts for monitoring nonconformities per unit.

Ryan and Woodall (2010) improved the performance of existing Poisson cumulative sum and EWMA charts using variable sample sizes. Shen et al. (2013) proposed the dynamic probability control limits for the EWMA chart when monitoring the Poisson count data. The Poisson adaptive EWMA (PAEWMA-I) charts were developed by Aly, Saleh and Mahmoud (2022, 2021) for tracking Poisson and zero-inflated Poisson count data, respectively. Similar to this, Abbas et al. (2023) proposed the unbiased function-based Poisson adaptive EWMA (PAEWMA-II) chart for monitoring the mean of the Poisson process. For more literature on Poisson control charts, interested readers are referred to Abbasi (2017), Khoo (2004), Li, Zhou and Ding (2020), and Sheu et al. (2023).

To increase the sensitivity of the EWMA scheme for each smoothing constant, Abbas et al. (2020) and Ali et al. (2021) recently applied EWMA statistic as an input in progressive setup (EWMA-p). This study proposes PEWMA statistic under progressive setup (PEWMA-p) to enhance the performance of the PEWMA scheme studied by Borror, Champ and Rigdon (1998) at all choices of weighting constants. The run-length (RL) profiles of the proposed PEWMA-p design have been computed using extensive Monte Carlo simulations.

The rest of the paper has been organized as; Methodology of the proposed and existing schemes are provided in the next section. The RL profiles are computed and compared with the counterparts in subsequent section. The application of the proposed scheme is reported in the following section. The study ends with a summary and concluding remarks.

METHODOLOGY

The mathematical modelings for the existing PEWMA, PEWMA-1, PEWMA-II, and proposed PEWMA-p charts are given briefly in this section. Consider X_t be the sequence of t independent Poisson observations for $X_t \geq 1$ with equal mean and variance, denoted by μ. For X_t the Poisson probability distribution function (PDF) is defined as:

$$f(X_t;\mu) = \frac{\mu^{X} e^{-X}}{X!}\,; X_t = 0,1,2,\ldots,\infty. \quad (1)$$

Based on X_t represents independent observations, in the next subsections the plotting statistics for the existing and proposed chats with their control limits are discussed. For the IC process; $X_t \sim Poisson\ (\mu_0)$ and for OOC process; $X_t \sim Poisson\ (\mu_1)$, where $\mu_1 = \mu_0 \pm \delta\sqrt{\mu_0}$ and μ_0 is a parameter of the IC Poisson parameter.

DESIGN OF PEWMA CHART

Consider E_t be the plotting statistics of the PEWMA chart based on X_t, suggested by Borror, Champ and Rigdon (1998);

$$E_t = \lambda X_t + (1 - \lambda)\, E_{t-1.} \quad (2)$$

where the weighting constant holds constants; $0 < \lambda \leq 1$. The control limits of the PEWMA chart with an initial value $E_0 = \mu_0$; where μ_0 is the specific parameter of the Poisson distribution.

$$\left.\begin{aligned} LCL &= \mu_0 - L\sqrt{\frac{\mu_0\lambda}{2-\lambda}}, \\ CL &= \mu_0, \\ UCL &= \mu_0 + L\sqrt{\frac{\mu_0\lambda}{2-\lambda}}. \end{aligned}\right\} \quad (3)$$

where L is a positive constant that determines the IC ARL ($ARL_0 \approx$ assumed 500) of the PEWMA chart. An OOC signal is activated by the PEWMA chart as soon as $E_t < LCL$ or $E_t > UCL$.

DESIGN OF PAEWMA-I CHART

The PAEWMA-I chart combines the characteristics of the Shewhart and PEWMA charts with an appropriate score mechanism based on current error $\zeta(e_t)$. The PEWMA-I chart can target a range of mean shift sizes because of this feature. Based on X_t, let's assume that the standardized form of the Poisson random variable can be expressed as; $Z_t = \frac{X_t-\mu_0}{\sqrt{\mu_0}}$. The plotting statistic of the PAEWMA-I chart based on Z_t is written by

$$D_t = D_{t-1} + \zeta(e_t) \tag{4}$$

where D_{t-1} is the preceding PAEWMA-I chart statistics at $t-1$ time, and the term e_t is the error at time t. Such that $e_t = Z_t - D_{t-1}$ and $\zeta(e_t)$ be the score function as defined by Capizzi and Masarotto (2003) as follows;

$$\zeta(e_t) = \begin{cases} e + (1-\gamma)\kappa & if\ e < -\kappa, \\ \gamma e & if\ |e| \leq \kappa, \\ e - (1-\gamma)\kappa & if\ e > \kappa. \end{cases} \tag{5}$$

where in (5) y and κ are the design parameters define their limits as $0 < \gamma \leq 1$ and $\kappa \geq 0$ respectively.The PAEWMA-I design alarms if the plotting statistic defined in (4) crosses the threshold (h).

DESIGN OF PAEWMA-II CHART

Recently, Abbas et al. (2023) investigated the PAEWMA-II chart based on an unbiased function for a range of shifts. Based on X_t, let's assume $Z_t = \frac{X_t-\mu_0}{\sqrt{\mu_0}}$, be the plotting-statistic of the PAEWMA-II chart is provided by;

$$U_t = \varphi Z_t + (1-\varphi)\, U_{t-1.} \tag{6}$$

where weight $0 < \varphi \leq 1$ is the smoothing parameter. The control limits of the PEWMA-II chart with an initial value $U_0 = 0$. The PAEWMA-II chart alarms the OOC signal if the plotting statistic given in (6) crosses the threshold (h).

DESIGN OF THE PROPOSED PEWMA-p CHART

Based on X_t, let's assume P_t be the plotting statistic of the proposed PEWMA-p chart represented as;

$$\begin{gathered} B_t = \alpha X_t + (1-\alpha)\, B_{t-1}. \\ P_t = \sum_{i=1}^{t} B_i \big/ t. \end{gathered} \tag{7}$$

where weight $0 < a \leq 1$ is the smoothing parameter. For the classical approach complete derivation of the mean and variance of the proposed PEWMA-p chart is given in Appendix B, and finally, the control limits are as follows,

$$\left.\begin{aligned} LCL_t &= \mu_0 - L\sqrt{\frac{\mu_0}{t}\left[1 + \frac{\eta^2}{t}\left(\frac{1-\eta^{2t}}{1-\eta^2}\right) - \frac{2\eta}{t}\left(\frac{1-\eta^t}{\alpha}\right)\right]} \\ CL &= \mu_0 \\ UCL_t &= \mu_0 + L\sqrt{\frac{\mu_0}{t}\left[1 + \frac{\eta^2}{t}\left(\frac{1-\eta^{2t}}{1-\eta^2}\right) - \frac{2\eta}{t}\left(\frac{1-\eta^t}{\alpha}\right)\right]} \end{aligned}\right\}. \tag{8}$$

where in (8), L represents the coefficient of control limits selected at the designated ARL_0 values and $\eta = 1 - \alpha$. If the charting scheme expressed in (7) crosses the control limits specified in Equation (8), the suggested PEWMA-p will activate the OOC signal.

RUN LENGTH EVALUATION AND DISCUSSION

The RL profiles of the proposed PEWMA-p chart have been computed in this section by applying Monte Carlo simulations. For the RL profiles of the proposed PEWMA-p chart, 100,000 iterations have been taken. Table 1 shows the estimated design parameters at $ARL_0 \approx 500$ and $\mu_0 = 1,4$, and 7, for the existing and the proposed charts. The Monte Carlo simulation algorithm of the proposed PEWMA-p chart includes the following steps: 1) Take a random sample of size $n = 1$, from the Poisson distribution with μ_0, 2) Take a smoothing parameter and compute the PEWMA-p plotting statistic, P_t given (7), 3) Take an arbitrary value of L and evaluate the control limits provided in (8), 4) Compare the proposed plotting statistic P_t, with control limits and note RLs, and 5) Compute the ARL_0 if it is equal to a nominal value of 500, choose the L set in step 3 otherwise repeat steps 1-5.

TABLE 1. The design parameters of the existing and proposed charts at IC $ARL_0 \approx 500$

μ_0	PEWMA		PAEWMA-I			PAEWMA-II		PEWMA-p	
	λ	K	γ	κ	h	φ	h	α	L
1	0.10	2.857	0.10	7.7403	0.6547	0.10	0.775	0.10	3.427
	0.25	3.287	0.25	7.7403	1.2427	0.25	1.284	0.25	3.588
4	0.10	2.824	0.10	7.7403	0.6595	0.10	0.75	0.10	3.4265
	0.25	3.062	0.25	7.7403	1.1695	0.25	1.128	0.25	3.5874
7	0.10	2.821	0.10	9.7699	0.661	0.10	0.729	0.10	3.4263
	0.25	3.028	0.25	9.7699	1.161	0.25	1.105	0.25	3.5873

At several small, moderate, and large shifts in the process setting $ARL_0 \approx 500$ corresponding to μ_0 = 1,4, and 7, ARL_1 profiles are estimated in Tables 2, 3, and 4. In Table 2 with μ_0 = 1, at α = 0.10 and δ = 0.25, the proposed PEWMA-p chart yields ARL_1 = 51.94 and the PEWMA, PAEWMA-I, and PAEWMA-II generate ARL_1 = 75.24, 74.85 & 53.63, respectively. Similarly, the proposed PEWMA-p chart has better shift detection ability as compared to the existing counterparts at all combinations of the design parameters (cf. Tables 2, 3, and 4).

APPLICATIONS

This section demonstrates how to implement both existing and proposed charts for detecting shifts in a Poisson process mean using an example. According to Aly, Saleh and Mahmoud (2021) and Batool and Haq (2024), imagine a fast-food establishment where, in typical situations, there are regularly 7 complaints from customers (X) each week on average. Furthermore, the distribution of (X) is Poisson, with an in-control mean of μ_0 = 7. The management of the restaurant is tolerant of this volume of complaints, and it has no negative effect on sales. The management of the restaurant started implementing various service enhancements to lower the number of weekly complaints. This progress was short-lived, nevertheless, as the restaurant unintentionally utilized a batch of low-quality pasta, which resulted in a sharp increase in weekly complaints. There have been periods when this spike has reached as high as 10 or more complaints.

Based on simulation, a data set of 50 observations or a sample that lasted 50 weeks is estimated from the Poisson distribution, which is defined by the parameter μ_0, to simulate this situation. The dataset has 30 samples in total;

TABLE 2. The ARL profile of the proposed and existing charts at μ_0 = 1 & $ARL_0 \approx 500$

δ	PEWMA		PAEWMA-I		PAEWMA-II		PEWMA-p	
	λ = 0.10	λ = 0.25	γ = 0.10	γ = 0.25	φ = 0.10	φ = 0.25	a = 0.10	a = 0.25
0.00	500.67	499.80	500.39	500.29	500.33	500.30	500.43	500.84
0.25	75.24	101.16	74.85	101.03	53.63	73.26	51.94	50.52
0.50	27.41	35.88	27.55	35.70	17.63	25.82	21.08	20.33
0.75	15.27	18.16	15.39	18.14	8.52	13.03	12.43	12.17
1.00	10.48	11.25	10.42	11.14	5.21	8.08	8.59	8.42
1.25	7.90	7.89	7.93	7.90	3.67	5.47	6.33	6.31
1.50	6.39	6.05	6.38	6.06	2.84	4.08	5.03	5.02
1.75	5.35	4.92	5.39	4.93	2.31	3.23	4.12	4.15
2.00	4.66	4.13	4.64	4.14	1.98	2.69	3.50	3.53
2.25	4.10	3.55	4.10	3.56	1.73	2.31	3.03	3.07
2.50	3.68	3.13	3.67	3.15	1.59	2.03	2.68	2.72
2.75	3.35	2.82	3.35	2.81	1.45	1.84	2.40	2.43
3.00	3.07	2.56	3.08	2.55	1.36	1.68	2.18	2.20
3.25	2.85	2.33	2.84	2.33	1.29	1.57	1.99	2.02
3.50	2.67	2.16	2.66	2.17	1.23	1.47	1.84	1.87
3.75	2.49	2.02	2.50	2.03	1.19	1.40	1.72	1.74
4.00	2.36	1.89	2.35	1.89	1.15	1.33	1.62	1.65
4.25	2.24	1.79	2.24	1.78	1.13	1.28	1.54	1.55
4.50	2.12	1.70	2.13	1.69	1.10	1.24	1.46	1.47
4.75	2.04	1.61	2.04	1.62	1.08	1.20	1.40	1.40
5.00	1.95	1.54	1.95	1.54	1.07	1.17	1.35	1.35
5.25	1.87	1.48	1.87	1.48	1.06	1.14	1.30	1.30
5.50	1.80	1.43	1.80	1.42	1.05	1.12	1.26	1.26
5.75	1.74	1.38	1.74	1.37	1.04	1.10	1.22	1.22
6.00	1.68	1.33	1.68	1.33	1.03	1.09	1.19	1.19

the first 20 are taken as the IC process (this means that μ_0 = 7), and the next 10 are obtained with a 1.5 process mean shift (this means that $\mu_0 + 1.5\sqrt{\mu_0}$). The observations are generated from the Poisson distribution for the PEWMA chart at design parameters $\lambda = 0.25$ and $K = 3.028$ in panel I of Figure 1. In Panel II of Figure 1 for PAEWMA-I chart at design parameters $\gamma = 0.25, \kappa = 9.7699$ and $h = 1.161$. In Panel III of Figure 1 for PAEWMA-II chart at design parameters $\varphi = 0.25$, and $H = 1.105$. In final panel IV of Figure 1 for the PEWMA-p chart at design parameters $\alpha = 0.25$ and $L = 3.5873$ at $ARL_0 \approx 500$.

In Figure 1, for the PEWMA and PAEWMA-I chart, we can see that the 39th value goes out of control with irregular variation and later some values become in control again. Also, an irregular variation can be seen in the PAEWMA-II chart after the shift. In the proposed chart less variation exists when either the process is in control or out of control. This chart also shows that the 39th value becomes out of control but after detecting the shift the process is smoothly out of control with a random pattern.

TABLE 3. The ARL profile of the proposed and existing charts at $\mu_0 = 4$ & $ARL_0 \approx 500$

δ	PEWMA		PAEWMA-I		PAEWMA-II		PEWMA-p	
	$\lambda = 0.10$	$\lambda = 0.25$	$\gamma = 0.10$	$\gamma = 0.25$	$\varphi = 0.10$	$\varphi = 0.25$	$a = 0.10$	$a = 0.25$
0.00	501.26	501.80	499.85	500.08	500.88	500.31	501.13	501.06
0.25	86.25	107.53	89.18	109.55	59.38	73.45	51.42	49.82
0.50	28.88	36.06	29.46	37.22	18.53	24.80	20.50	20.13
0.75	15.52	17.44	15.72	17.70	8.56	12.25	12.01	11.82
1.00	10.30	10.59	10.42	10.64	5.03	7.39	8.11	8.14
1.25	7.78	7.38	7.75	7.42	3.51	4.98	6.04	6.07
1.50	6.21	5.62	6.17	5.61	2.69	3.63	4.74	4.81
1.75	5.20	4.54	5.07	4.52	2.20	2.85	3.87	3.96
2.00	4.49	3.83	4.31	3.79	1.88	2.33	3.26	3.37
2.25	3.96	3.32	3.77	3.28	1.65	1.99	2.82	2.92
2.50	3.56	2.96	3.30	2.89	1.49	1.75	2.46	2.58
2.75	3.23	2.65	2.95	2.58	1.37	1.58	2.19	2.31
3.00	2.97	2.42	2.66	2.34	1.29	1.45	1.96	2.10
3.25	2.75	2.24	2.39	2.14	1.22	1.35	1.80	1.92
3.50	2.56	2.08	2.19	1.98	1.17	1.28	1.65	1.78
3.75	2.41	1.95	2.02	1.83	1.13	1.22	1.54	1.66
4.00	2.27	1.84	1.86	1.72	1.10	1.17	1.44	1.55
4.25	2.15	1.74	1.72	1.63	1.08	1.13	1.36	1.47
4.50	2.04	1.65	1.62	1.53	1.06	1.11	1.29	1.40
4.75	1.95	1.57	1.52	1.45	1.04	1.08	1.24	1.34
5.00	1.87	1.50	1.44	1.39	1.03	1.06	1.19	1.28
5.25	1.79	1.44	1.37	1.33	1.02	1.05	1.16	1.23
5.50	1.72	1.38	1.31	1.28	1.02	1.04	1.13	1.19
5.75	1.65	1.33	1.25	1.24	1.01	1.03	1.10	1.16
6.00	1.59	1.28	1.21	1.20	1.01	1.02	1.08	1.13

TABLE 4. The ARL profile of the proposed and existing charts at $\mu_0 = 7$ & $ARL_0 \approx 500$

δ	PEWMA		PAEWMA-I		PAEWMA-II		PEWMA-p	
	$\lambda = 0.10$	$\lambda = 0.25$	$\gamma = 0.10$	$\gamma = 0.25$	$\varphi = 0.10$	$\varphi = 0.25$	$a = 0.10$	$a = 0.25$
0.00	501.26	498.19	500.23	501.37	500.06	500.73	498.58	501.78
0.25	90.08	114.37	93.63	118.98	59.50	74.68	50.74	50.04
0.50	29.37	37.82	30.13	39.14	18.35	24.63	20.48	19.99
0.75	15.49	17.63	15.87	18.31	8.13	11.83	11.89	11.72
1.00	10.34	10.54	10.41	10.77	4.73	7.04	8.07	8.05
1.25	7.72	7.30	7.75	7.38	3.22	4.67	6.00	6.01
1.50	6.19	5.53	6.12	5.56	2.46	3.39	4.72	4.74
1.75	5.18	4.45	5.03	4.45	1.96	2.63	3.84	3.88
2.00	4.48	3.74	4.29	3.73	1.66	2.16	3.25	3.29
2.25	3.93	3.25	3.70	3.20	1.47	1.83	2.80	2.84
2.50	3.54	2.87	3.24	2.79	1.33	1.62	2.46	2.50
2.75	3.21	2.59	2.86	2.50	1.23	1.45	2.20	2.23
3.00	2.96	2.37	2.57	2.25	1.17	1.35	1.99	2.02
3.25	2.74	2.19	2.33	2.05	1.12	1.27	1.83	1.85
3.50	2.57	2.04	2.10	1.88	1.08	1.20	1.69	1.70
3.75	2.41	1.91	1.92	1.74	1.06	1.15	1.58	1.59
4.00	2.29	1.80	1.77	1.62	1.04	1.12	1.48	1.49
4.25	2.18	1.71	1.63	1.53	1.03	1.09	1.40	1.39
4.50	2.08	1.62	1.51	1.44	1.02	1.06	1.32	1.33
4.75	2.00	1.54	1.41	1.37	1.01	1.05	1.27	1.27
5.00	1.92	1.47	1.34	1.30	1.01	1.04	1.22	1.22
5.25	1.86	1.41	1.27	1.24	1.01	1.02	1.17	1.17
5.50	1.79	1.35	1.21	1.20	1.00	1.02	1.14	1.14
5.75	1.72	1.30	1.17	1.16	1.00	1.01	1.11	1.11
6.00	1.66	1.25	1.14	1.13	1.00	1.01	1.09	1.08

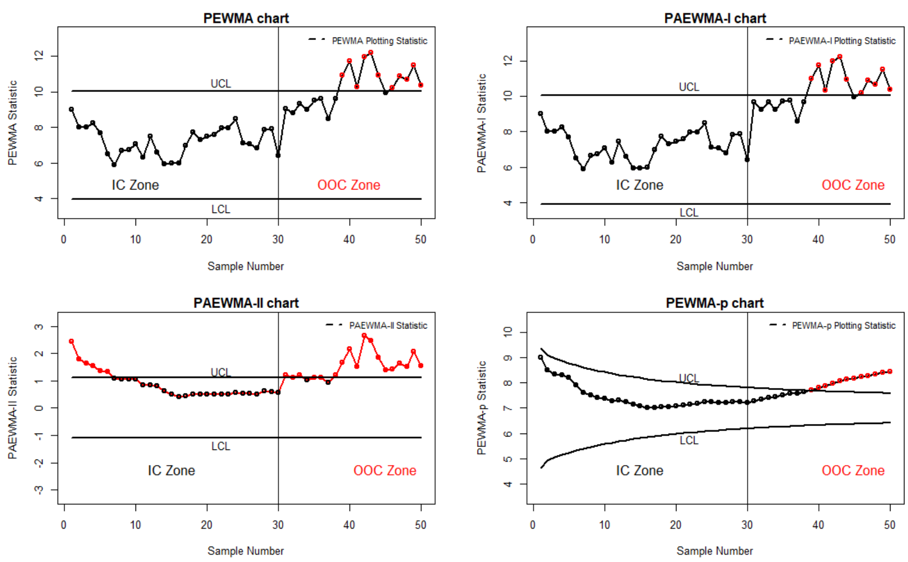

FIGURE 1. Application of existing and proposed PEWMA-p chart at $ARL_0 \approx 500$

CONCLUSION

Control chart structures have undergone numerous modifications by quality practitioners to identify sudden and sequential shifts in ongoing processes quickly. The PEWMA-p chart has been suggested in this work for Poisson process monitoring. The proposal showed that it could efficiently identify a variety of mean shifts. The RL profiles of the proposed and existing charts have been determined using Monte Carlo simulations. The findings showed that the PEWMA-p chart performs better than the existing competitors; PEWMA, PAEWMA-I, and PAEWMA-II charts at small to moderate shifts. Additionally, an illustrative application related to restaurant management has been included to improve the quality of food. The proposal can be extended to unknown cases of process parameters and multivariate attribute control charts.

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APPENDIX A

Derivation of mean and variance of the plotting statistic B_t.

$B_t = aX_t + (1 - a) B_{t-1}$

$B_t = aX_t + (1 - a) [aX_{t-1} + (1 - a)B_{t-2}]$

$B_t = aX_t + (1 - a) aX_{t-1} + (1 - a)^2 B_{t-2}]$

$B_t = aX_t + (1 - a)aB_{t-1} + (1 - a)^2 [aX_{t-2} + (1 - a)B_{t-3}]$

$B_t = aX_t + (1 - a)aB_{t-1} + (1 - a)^2 [aX_{t-2} + (1 - a)^3 B_{t-3}$

…

$$B_t = \alpha \sum_{i=0}^{t-1} (1-\alpha)^i X_{t-i} + (1-\alpha)^t B_0$$

(A1)

To estimate the mean of B_t after applying expectations on both sides of (A1) we get;

$$E(B_t) = \alpha \sum_{i=0}^{t-1} (1-\alpha)^i E(X_{t-i}) + (1-\alpha)^t E(B_0)$$

Therefore $E(X_{t-1}) = \mu_0$ and $E(B_0) = E(X_0) = \mu_0$, so we get.

$$E(B_t) = \alpha\mu_0 \sum_{i=0}^{t-1} (1-\alpha)^i + (1-\alpha)^t (\mu_0)$$

$E(B_t) = \alpha\mu_0 [(1 - \alpha)^0 + (1 - \alpha)^1 + (1 - \alpha)^2 + ... + (1 - \alpha)^{t-1}] + (1 - \alpha)^t (\mu_0)$

$E(B_t) = \alpha\mu_0 [1 + (1 - \alpha)^1 + (1 - \alpha)^2 + ... + (1 - \alpha)^{t-1}]$

$$E(B_t) = \alpha\mu_0 \left[\frac{1}{1-(1-\alpha)}\right]$$

$$E(B_t) = \alpha\mu_0 \left[\frac{1}{\alpha}\right]$$

$E(B_t) = \mu_0$

(A2)

This is the required mean of the plotting statistic B_t, now for variance, apply variance on both sides of equation (A1).

$$var(B_t) = \alpha^2 \sum_{i=0}^{t-1} (1-\alpha)^{2i} var(X_{t-i}) + (1-\alpha)^t var(P_0)$$

Therefore $var(X_{t-1}) = \mu_0$ and $var(B_0) = \mu_0$, so we get.

$$var(B_t) = \alpha^2 \sum_{i=0}^{t-1} (1-\alpha)^{2i} \sigma_X^2$$

$$var(B_t) = \alpha^2 \mu_0 \left[\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2}\right]$$

$$var(B_t) = \frac{\alpha\mu_0}{2-\alpha}[1-(1-\alpha)^{2t}]$$

(A3)

This is the required variance of the plotting statistic.

APPENDIX B

Derivation of mean and variance of PEWMA-p.

The PEWMA-p statistic is:

$$P_t = \frac{\sum_{i=1}^t B_t}{t}$$

(B1)

For mean apply expectation on both sides of equation (B1)

$$E(P_t) = E\left(\frac{\sum_{i=1}^t B_t}{t}\right)$$

$$E(P_t) = \frac{1}{t}\sum_{i=1}^t E(B_t)$$

$$E(P_t) = \frac{1}{t}\sum_{i=1}^t \mu_0$$

$$E(P_t) = \frac{1}{t} t\mu_0$$

$E(P_t) = \mu_0$ (B2)

This is the required mean, now for variance use equation (B1) again and replace P_t from equation (A1).

$$P_t = \frac{\sum_{i=1}^t B_t}{t}$$

$$P_t = \frac{1}{t}\sum_{l=1}^t \left(\alpha \sum_{i=0}^{l-1} (1-\alpha)^i X_{t-i} + (1-\alpha)^t B_0\right)$$

$$P_t = \frac{\alpha}{t}\sum_{l=1}^t \sum_{i=0}^{l-1} (1-\alpha)^i X_{t-i} + \frac{1}{t}\sum_{l=1}^t (1-\alpha)^t B_0$$

After applying variance to the above equation, we get

$$var(P_t) = var\left(\frac{\alpha}{t}\sum_{l=1}^t \sum_{i=0}^{l-1} (1-\alpha)^i X_{t-i} + \frac{1}{t}\sum_{l=1}^t (1-\alpha)^t B_0\right)$$

$$var(P_t) = \left(\frac{\alpha}{t}\sum_{l=1}^t \sum_{i=0}^{l-1} (1-\alpha)^i\right)^2 var(X_{t-i}) + \left(\frac{1}{t}\sum_{l=1}^t (1-\alpha)^t\right)^2 var(B_0)$$

Therefore $var(X_{t-1}) = \mu_0$ and $var(B_0) = \mu_0$, so we get.

$$var(P_t) = \left(\frac{\alpha}{t}\sum_{l=1}^t \sum_{i=0}^{l-1} (1-\alpha)^i\right)^2 \mu_0$$

$$var(P_t) = \mu_0 \frac{\alpha^2}{t^2}\left(\sum_{l=1}^t \sum_{i=0}^{l-1} (1-\alpha)^i\right)^2$$

$$var(P_t) = \frac{\alpha^2 \mu_0}{t^2}\left(\sum_{l=1}^{t} \frac{1-(1-\alpha)^{t-l+1}}{\alpha}\right)^2$$

$$var(P_t) = \frac{\mu_0}{t^2}\sum_{l=1}^{t}(1-(1-\alpha)^{t-l+1})^2$$

$$var(P_t) = \frac{\mu_0}{t^2}\sum_{l=1}^{t}\left(1+(1-\alpha)^{2(t-l+1)}-2(1-\alpha)^{t-l+1}\right)^2$$

After applying the geometric progression will be

$$var(P_t) = \frac{\mu_0}{t^2}\left[\sum_{l=1}^{t} 1 + \sum_{l=1}^{t}(1-\alpha)^{2(t-l+1)} - 2\sum_{l=1}^{t}(1-\alpha)^{(t-l+1)}\right]$$

$$var(P_t) = \frac{\mu_0}{t^2}\left[t + (1-\alpha)^2\left(\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2}\right) - 2(1-\alpha)\left(\frac{1-(1-\alpha)^t}{\alpha}\right)\right]$$

Let $\eta = 1 - \alpha$, after replacing we get

$$var(P_t) = \frac{\mu_0}{t^2}\left[t + \eta^2\left(\frac{1-\eta^{2t}}{1-\eta^2}\right) - 2\eta\left(\frac{1-\eta^t}{\alpha}\right)\right]$$

$$var(P_t) = \frac{\mu_0}{t}\left[1 + \frac{\eta^2}{t}\left(\frac{1-\eta^{2t}}{1-\eta^2}\right) - \frac{2\eta}{t}\left(\frac{1-\eta^t}{\alpha}\right)\right]$$

This is the required variance.