Sains Malaysiana 55(10)(2026): 1610-1617
http://doi.org/10.17576/jsm-2026-5510-01

Zagreb Index of Fuzzy Prime Coprime Graph of Integers Modulo Groups

(Indeks Zagreb daripada Graf Koprime Perdana Kabur bagi Kumpulan Integer Modulo)

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Received: 27 July 2025/Accepted: 10 September 2026

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ABSTRACT

This study investigates the structural properties and topological indices of fuzzy prime coprime graphs constructed from integer modulo groups. A fuzzy prime coprime graph generalizes the conventional prime coprime graph by assigning membership values in the interval [0,1] to both vertices and edges, reflecting varying degrees of association. Unlike previous studies that used arbitrary assignments, this research introduces a systematic and mathematically rigorous method of weighting based on the greatest common divisor (gcd) and least common multiple (lcm) of the orders of group elements. The main contributions include the formulation of membership functions, the classification of vertex and edge types, and the derivation of closed-form expressions for the first Zagreb, second Zagreb, and entire Zagreb indices of the fuzzy graph. These results provide new insights into the interaction between fuzzy set theory and algebraic graph structures, promoting advances in mathematical modeling, fuzzy logic systems, and computational methods for complex networks. The approach also supports future research in discrete mathematics, data-driven systems, and symbolic computation.

Keywords: Fuzzy prime coprime graph; group $\mathbb{Z}_n$; Zagreb index

ABSTRAK

Penyelidikan ini mengkaji sifat struktur dan indeks topologi bagi graf koprime perdana kabur yang dibina daripada kumpulan modulo integer. Graf koprima perdana kabur menyamaratakan graf koprima perdana konvensional dengan memberikan nilai keahlian dalam selang [0,1] kepada kedua-dua bucu dan tepi, mencerminkan darjah perkaitan yang berbeza-beza. Tidak seperti kajian terdahulu yang menggunakan tugasan sewenang-wenangnya, penyelidikan ini memperkenalkan kaedah pemberat yang sistematik dan teliti secara matematik berdasarkan pembahagi sepunya terbesar (gcd) dan gandaan sepunya terkecil (lcm) bagi susunan unsur kumpulan. Sumbangan utama termasuk perumusan fungsi keahlian, pengkelasan jenis bucu dan tepi dan terbitan ungkapan bentuk tertutup untuk Zagreb pertama, Zagreb kedua dan keseluruhan indeks Zagreb bagi graf kabur. Keputusan ini memberikan pandangan baharu tentang interaksi antara teori set kabur dan struktur graf algebra, menggalakkan kemajuan dalam pemodelan matematik, sistem logik kabur dan kaedah pengiraan untuk rangkaian kompleks. Pendekatan ini juga menyokong penyelidikan masa depan dalam matematik diskret, sistem dipacu data dan pengiraan simbolik.

Kata kunci: Graf koprime perdana kabur; indeks Zagreb; kumpulan $\mathbb{Z}_n$

INTRODUCTION

Fuzzy set was first pioneered in 1965 by a professor at the University of California, Berkeley, United States, Lotfi A. Zadeh. In his paper entitled 'Fuzzy Sets', Zadeh generalized the concept of classical sets. In classical set theory, a set is defined as a collection of crisp objects, where each object can be determined with certaincty whether it belongs to the set or not. The set A in a universe X can be defined by the function C_A: X → {0, 1}, which is called the characteristic function of the set A. By developing the concept of characteristic function, a fuzzy set is defined through membership functions whose values are in the closed interval [0, 1] (Zadeh 1965). Thus, membership in a fuzzy set is no longer binary as in a classical set, but has a value of membership in the interval [0, 1].

Fuzzy set theory has since been integrated with numerous mathematical structures, including algebraic systems and graph theory (Islam & Pal 2023; Romdhini et al. 2023). In algebra, for instance, fuzzy concepts have been applied to the study of semigroups and ideals (Wang et al. 2024). In graph theory, the notion of fuzzy graphs,

originally introduced to model imprecise relationships, has led to active research on their structural and numerical properties (Al-Mutab 2019). This study combines fuzzy graphs with graph topological indices. Several studies have examined the topological indices of fuzzy graphs; for example, Kalathian et al. (2020) discussed several topological indices of fuzzy graphs. Fayazi, Mahmudi and Gholami (2020) studied the concept of the first and second Zagreb, Wiener, hyper-Wiener, and reverse-Wiener indices of fuzzy line and star graphs. Mufti et al. (2023a) discussed the topological indices of fuzzy pizza graphs. In the same year, Mufti et al. (2023b) also presented the topological indices on the *qCn* graph (Snake Graph). Other studies have explored the representation of mathematical systems, such as groups and rings, using graph structures. These include various types of algebraic graphs, such as coprime graphs (Nurhabibah, Wardhana & Switrayni 2023) and non-coprime graphs (Afdhaluzzikri et al. 2025). Each captures specific algebraic relationships between elements within a structure.

Taking ideas from several previous articles, the graphs used in those articles are special graphs such as line, star, pizza, and snake graphs. Therefore, in this article, we will discuss the prime coprime graphs, which have been studied by Adhikari and Banerjee (2022), and their spectral structures can be found in Romdhini et al. (2025a, 2025b). Related studies on co-prime order graphs were conducted by Sehgal, Manjeet and Singh (2021) for finite Abelian and dihedral groups, Hao, Zhong and Ma (2022) for general group properties, and Saini et al. (2025) for degree-based matrices. Furthermore, several indices have been studied on this graph (Abdurahim, Romdhini & Husain 2025; Abdurahim et al. 2025). In previous fuzzy graph studies, membership values were generally assigned using prescribed weighting schemes that were not explicitly derived from the underlying algebraic structure. In contrast, this study determines vertex and edge membership values from the orders of group elements using the greatest common divisor (gcd) and least common multiple (lcm).

The novelty and main contributions of this study lie in the construction of a fuzzy prime coprime graph with algebraically determined membership functions. Furthermore, closed-form expressions for selected topological indices are established. Thus, the proposed framework provides a mathematical connection between the algebraic structure of finite groups and the topological invariants of fuzzy graphs.

PRELIMINARIES

In this study, the Prime Coprime Graph refers to the graph introduced by Adhikari and Banerjee (2022), which is defined as follows.

Definition 2.1 Let G be a finite group where $|G| > 2$. The prime coprime graph of G, denoted by Γ_G, is a graph whose vertex set is $V(\Gamma_G) = G$. Two distinct vertices $x, y \in G$ are adjacent if and only if the greatest common divisor of their orders is equal to 1 or is a prime number; that is $gcd(|x|, |y|) = 1$ or $gcd(|x|, |y|)$ is a prime number.

Fuzzy graph of a graph $G = (V(G), E(G))$ is a pair of functions denoted by $\tilde{G} = (\sigma, \mu)$, where $\sigma : V(G) \rightarrow [0, 1]$ and $\mu: V(G) \times V(G) \rightarrow [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for every $u, v \in V(G)$ and $\sigma(u) \wedge \sigma(v) = \min\{\sigma(u), \sigma(v)\}$ (Al Mutab 2019).

The degree of a vertex v in a fuzzy graph is the sum of the weights of the edges incident to v and is denoted by $d(v)$ and defined as $d(v) = \Sigma_{uv \in E} \mu(u, v)$ with $u \neq v$ (Al Mutab 2019). In the same manner, the degree of an edge uv in a fuzzy graph is the sum of weights of edges incident on the vertices u and v in the edge. The degree of edge is denoted by $d(u, v)$ and defined as $d(u, v) = \Sigma_{xu \in E} \mu(xu) + \Sigma_{yv \in E} \mu(yv)$ with $x \neq u$ and $y \neq v$.

Example 2.1 Let $\Gamma_{\mathbb{Z}_9}$ be the prime coprime graph of $\mathbb{Z}_9$ and the membership value of the vertices is $\sigma(0) = 1.00$, $\sigma(1) = \sigma(2) = \sigma(4) = \sigma(5) = \sigma(7) = \sigma(8) = 0.11$, and $\sigma(3) = \sigma(6) = 0.33$. In addition, the membership value of the edge is $\mu(u, v) = 0.11$, for all $u, v \in V(\Gamma_{\mathbb{Z}_9})$ excluding $\mu(0, 3) = \mu(0, 6) = \mu(3, 6) = 0.33$. Thus, we obtain the fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_9}}$ as presented in the following Figure 1.

Moreover, the degree of every vertex in $\widetilde{\Gamma_{\mathbb{Z}_9}}$ is $d(0) = d(3) = d = 6 = 1.32$ and $d(1) = d(2) = d(4) = d(5) = d(7) = d(8) = 0.33$.

Example 2.2 Using the same explanation as in Example 2.1, for the prime coprime graph $\Gamma_{\mathbb{Z}_{16}}$, the membership degrees of all vertices are $\sigma(0) = 1.00$, $\sigma(1) = \sigma(2) = \ldots = \sigma(7) = \sigma(9) = \sigma(10) = \ldots = \sigma(15) = 0.0625$, and $\sigma(8) = 0.500$. Moreover, the membership degree of each edge is $\mu(u, v) = 0.03$, for all $u, v \in V(\Gamma_{16})$ except $\mu(0, 8) = 0.500$. Therefore, the corresponding fuzzy prime coprime graph $\widetilde{\Gamma_{\mathbb{Z}_{16}}}$ is obtained as follows.

Furthermore, the degree of each vertex in the fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_{16}}}$ is given by $d(0) = d(8) = 1.34$ and $d(1) = d(2) = \ldots d(7) = d(9) = d(10) = \ldots d(15) = 0.0625$.

Definition 2.2 (Islam & Pal 2021) Let $\widetilde{\Gamma_{\mathbb{Z}_n}} = (\sigma, \mu) =$ be the fuzzy graph of a prime coprime graph $\Gamma_{\mathbb{Z}_n}$ for $n = p^k$ where p prime and integer $k \geq 2$. The first Zagreb index of the fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_n}}$ is denoted by $M_1(\widetilde{\Gamma_{\mathbb{Z}_n}})$, and defined as

$$M_1(\widetilde{\Gamma_{\mathbb{Z}_n}}) = \Sigma_{u \in V(\Gamma_{\mathbb{Z}_n})}(\sigma(v) \cdot d(v))^2$$

with $d(v)$ and $\sigma(v)$ are the degree and membership value of a vertex v, respectively.

Example 2.3 From Example 2.1, we get the first Zagreb index of the fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_9}}$ as follows.

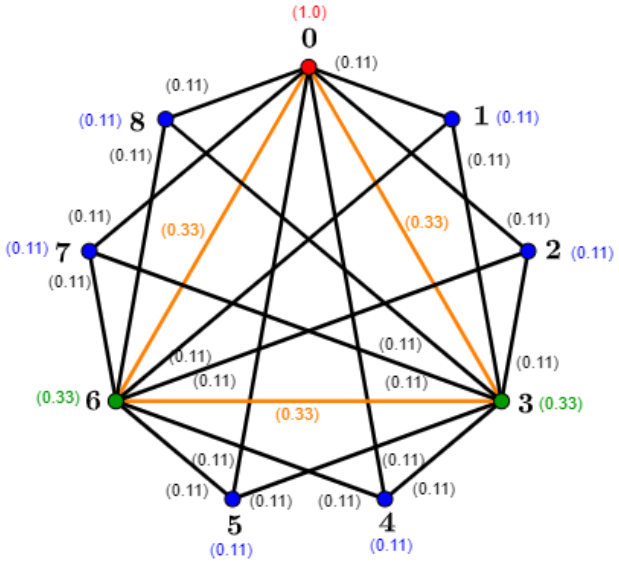


FIGURE 1. Fuzzy graph for $\Gamma_{\mathbb{Z}_9}$

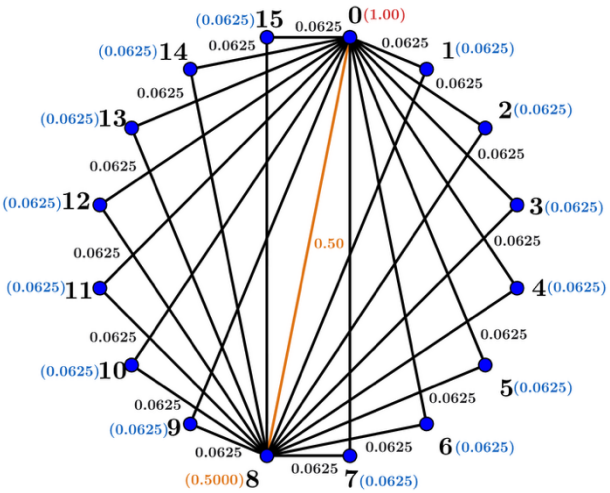


FIGURE 2. Fuzzy graph for $\Gamma_{\mathbb{Z}_{16}}$

$M_1(\widetilde{\Gamma_{\mathbb{Z}_9}}) = (\sigma(0)d(0))^2 + (\sigma(1)d(1))^2 + (\sigma(2)d(2))^2 + (\sigma(3)d(3))^2 + (\sigma(4)d(4))^2 + (\sigma(5)d(5))^2 + (\sigma(6)d(6))^2 + (\sigma(7)d(7))^2 + (\sigma(8)d(8))^2$
$= (1.00\cdot 1.33)^2 + (0.11\cdot 0.33)^2 + (0.11\cdot 0.33)^2 + (0.33\cdot 1.33)^2 + (0.11\cdot 10.33)^2 + (0.11\cdot 0.33)^2 + (0.33\cdot 1.33)^2 + (0.11\cdot 0.33)^2 + (0.11\cdot 0.33)^2$

$M_1(\widetilde{\Gamma_{\mathbb{Z}_9}}) = 2.1811.$

Thus, the first Zagreb of a fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_9}}$ is 2.1811.

Definition 2.3 (Tabraiz et al. 2023) Let $\widetilde{\Gamma_{\mathbb{Z}_n}} = (\sigma, \mu)$ be a fuzzy graph of prime coprime graph $\Gamma_{\mathbb{Z}_n}$ for $n = p^k$ where p prime and integer $k \geq 2$. The second Zagreb of fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_n}}$ is denoted by $M_2(\widetilde{\Gamma_{\mathbb{Z}_9}})$ and defined as

$$M_2(\widetilde{\Gamma_{\mathbb{Z}_n}}) = \tfrac{1}{2}\Sigma_{uv\in E(\Gamma_{\mathbb{Z}_n})}\sigma(u)d(u)\sigma(v)d(v)$$

where $d(u)$ and $d(v)$ are the degree of u and v, respectively.

Example 2.4 Considering Example 2.1, we can obtain the second Zagreb index of a fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_9}}$ as given below.

$M_2(\widetilde{\Gamma_{\mathbb{Z}_9}}) = \frac{1}{2}(\sigma(0)d(0)\sigma(1)d(1) + \sigma(0)d(0)\sigma(2)d(2) + \sigma(0)d(0)\sigma(3)d(3) + \sigma(0)d(0)\sigma(4)d(4) + \sigma(0)d(0)\sigma(5)d(5) + \sigma(0)d(0)\sigma(6)d(6) + \sigma(0)d(0)\sigma(7)d(7) + \sigma(0)d(0)\sigma(8)d(8) + \sigma(1)d(1)\sigma(3)d(3) + \sigma(1)d(1)\sigma(6)d(6) + \sigma(2)d(2)\sigma(3)d(3) + \sigma(2)d(2)\sigma(6)d(6) + \sigma(3)d(3)\sigma(4)d(4) + \sigma(3)d(3)\sigma(5)d(5) + \sigma(3)d(3)\sigma(7)d(7) + \sigma(3)d(3)\sigma(8)d(8) + \sigma(4)d(4)\sigma(6)d(6) + \sigma(5)d(5)\sigma(6)d(6) + \sigma(6)d(6)\sigma(7)d(7) + \sigma(6)d(6)\sigma(8)d(8))$

$$M_2(\widetilde{\Gamma_{\mathbb{Z}_9}}) = 0.9383.$$

Therefore, the second Zagreb index of a fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_9}}$ is 0.9383.

Definition 2.4 (Jana & Ghorai 2023) Let $\widetilde{\Gamma_{\mathbb{Z}_n}} = (\sigma, \mu)$ be the fuzzy graph of a prime coprime graph $\widetilde{\Gamma_{\mathbb{Z}_n}}$ for $n = p^k$ where p prime and integer $k \geq 2$. The entire Zagreb index of a fuzzy graph $\widetilde{\Gamma_{\mathbb{Z}_n}}$ is denoted by $M_e(\widetilde{\Gamma_{\mathbb{Z}_n}})$ and defined as

$M_e(\widetilde{\Gamma_{\mathbb{Z}_n}}) = \Sigma_{u \in V(\Gamma_{\mathbb{Z}_n})} (\sigma(u)d(u))^2 + \Sigma_{uv \in E(\Gamma_{\mathbb{Z}_n})} (\mu(u, v)d(u, v))^2$ where d(u), μ(u, v), and d(u, v) are the degree of u, weight, and degree of edge uv, respectively.

In Definition 2.4, the first Zagreb index implies calculating the value of the entire Zagreb index, we must indirectly calculate the value of the first Zagreb index.

Theorem 2.1 (Abdurahim et al. 2026) Let $\Gamma_{\mathbb{Z}_n}$ be the prime coprime graph of $\mathbb{Z}_n$. If $n = p^k$ for p prime and integer $k \geq 2$, then the subgraph of $\Gamma_{\mathbb{Z}_n}$ is a complete graph K_p and a bipartite graph $K_{p, p^k - p}$.

Theorem 2.2 (Abdurahim et al. 2026) Let $\Gamma_{\mathbb{Z}_n}$ be the prime coprime graph of $\mathbb{Z}_n$. If $n = p^k$ for p prime and integer $k \geq 2$, then the vertex degree in $\Gamma_{\mathbb{Z}_n}$ is

$$\deg(v) = \begin{cases} p^k - 1 & , v \in V_1 \\ p & , v \in V_2, \end{cases}$$

where $V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1}, \ldots, (p-1)p^{k-1}\}$ and $V_2 = \{0, 1, 2, 3, \ldots, p^{k-1}\} \setminus V_1$.

MAIN RESULTS

In this section, we first define the membership value of vertices and edges on the fuzzy graph. Let $\Gamma_{\mathbb{Z}_n}$ be the prime coprime graph. The membership value of a vertex is defined as follows:

$$\sigma(u) = \begin{cases} \frac{1}{|u|} & , |u| = 1 \text{ or prime} \\ \frac{1}{n} & , \text{otherwise.} \end{cases}$$

and the membership value of an edge is

$$\mu(u, v) = \begin{cases} \frac{1}{lcm(|u|, |v|)} & , |u|, |v| = 1 \text{ or prime} \\ \frac{1}{n} & , \text{otherwise.} \end{cases}$$

where $|u|$ and $|v|$ are orders of u and v.

The fuzzy membership values assigned to the vertices in the fuzzy graph of the prime coprime graph are classified into three distinct categories. This classification is formalized in the following theorem.

Definition 3.1 Let $\Gamma = (V(\Gamma), E(\Gamma))$ be a prime coprime graph. A fuzzy prime coprime graph of Γ is defined as a pair of functions $\tilde{\Gamma} = (\sigma, \mu)$, where σ: $V(\Gamma) \rightarrow [0, 1]$ is the vertex membership function and μ: $V(\Gamma) \times V(\Gamma) \rightarrow [0, 1]$ is the edge membership function satisfying $\mu(u, v) \leq \sigma(u) \wedge \sigma(v) = \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V(\Gamma)$.

Theorem 3.1 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V, \sigma, \mu)$ be the fuzzy graph of the prime coprime graph $\Gamma_{\mathbb{Z}_{pk}}$ where $k \geq 2$ are integers, then the membership value of the vertex in $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$ is as follows:

(i) $\sigma(0) = 1$;

(ii) $\sigma(u) = \frac{1}{p}$ for $u \in V_1$;

(iii) $\sigma(v) = \frac{1}{p^k}$ for $u \in V_2$,

where $V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1}, \ldots, (p-1)p^{k-1}\}$ and $V_2 = \{0, 1, 2, 3, \ldots, p^k - 1\} \setminus V_1$.

Proof

Based on the proof of Theorem 2.2, we have $|w| = 1$, $|u| = p$, and $|v| = r.p$ for $w = 0$, $u \in V_1$ and $v \in V_2$. Furthermore, by the properties of the membership value of a vertex $\sigma(u)$, it can be seen that $\sigma(0) = 1$ $\sigma(u) = \frac{1}{p}$, and $\sigma(v) = \frac{1}{p^k}$.

In addition to the fuzzy membership values of the vertices, there are also fuzzy membership values assigned to the edges of the graph. These values are categorized into two types. This is formalized in the following theorem.

Theorem 3.2 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V\ \sigma, \mu)$ be the fuzzy graph of the prime coprime graph $\Gamma_{\mathbb{Z}_{pk}}$ where $k \geq 2$ are integers, then the membership value of an edge in $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$ is as follows:

(i) $\mu(u_1, v_1) = \frac{1}{p}$;

(ii) $\mu(u_1, v_2) = \frac{1}{p^k}$,

for $u_1, v_1 \in V_1$ and $v_2 \in V_2$, where $V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1}, \ldots, (p-1)p^{k-1}\}$ and $V_2 = \{0, 1, 2, 3, \ldots, p^k - 1\} \setminus V_1$.

Proof

Let $u_1\ v_1 \in \{0\} V_1$ and $v_2 \in V_2$. According to the proof of Theorem 3.1, we get either $|u_1| = 1$ or p, either $|v_1| = 1$ or p, and $|v_1| = r \cdot p$ for r is a natural number. Consequently, the least common multiple lcm $(u_1\ v_1) = p$ and lcm $(u_1\ v_2) = r \cdot p$. Based on the properties of the membership value of an edge, $\mu = (u_1\ v_1) = \frac{1}{p}$ and $\mu = (u_1\ v_2) = \frac{1}{n} = \frac{1}{p^k}$, and the proof is completed.

Next, theorems regarding the degree of vertices and edges in a fuzzy graph are presented in Theorems 3.3 and 3.4.

Theorem 3.3 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V\ \sigma, \mu)$ be the fuzzy graph of a prime coprime graph $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$ with $k \geq 2$ are integers, then the degree of the vertex in $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$ is as follows:

(i) $d(0) = d(u) = (p-1)\cdot\frac{1}{p} + (p^k - p)\cdot\frac{1}{p^k}$ for $u \in V_1$;

(ii) $d(v) = \frac{1}{p^k}$ for $v \in V_2$,

where $V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1},..., (p-1)p^{k-1}\}$, and $V_2 = \{0, 1, 2, 3,..., p^k - 1\} \setminus V_1$.

Proof

By Theorem 2.2, we know that there are two values of the vertex degree in the prime coprime graph, which are $\deg(u) = p^k - 1$ for $u \in V_1 \cup \{0\}$ and $\deg(v) = p$ for $v \in V_2$. According to Theorem 2.1, $\Gamma_{\mathbb{Z}_{pk}}$ with the vertex set $V_1 \cup \{0\}$ is a complete graph. Meanings that for $u \in V_1 \cup \{0\}$, $\Gamma_{\mathbb{Z}_{pk}}$ has $p-1$ incident edges in which there are $p-1$ edges $u_1 v_1$ and $p^k - p$ edges $u_1 v_2$ for $u_1, v_1 \in V_1 \cup \{0\}$ and $v_2 \in V_2$. Since $d(u)$ is the summation of the membership value of edges that are incident on the vertex u, and by Theorem 9, we have

$$d(u) = \sum_{uv \in V} \mu(u,v) = \sum_{u_1 v_1 \in V} \mu(u_1, v_1) + \sum_{u_1 v_2 \in V} \mu(u_1, v_2)$$

$$d(u) = (p-1)\cdot\frac{1}{p} + (p^k - p)\cdot\frac{1}{p^k}.$$

Moreover, based on Theorem 2.2, then $\deg(v) = p$ and from Theorem 3.2, $\mu = (u_1 v_2) = \frac{1}{p^k}$ then we have

$$d(v) = \sum_{u_1 v_2 \in V} \mu(u_1, v_2) = p \cdot \frac{1}{p^k} = \frac{p}{p^k}$$

Theorem 3.4 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V, \sigma, \mu)$ be the fuzzy graph of the prime coprime graph $\Gamma_{\mathbb{Z}_{pk}}$ with $k \geq 2$ are integers, then the degree of the edge in $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$ is

(i) $d(u_1, v_1) = 4 - \frac{4}{p} - \frac{2p}{p^k}$;

(ii) $d(u_1, v_2) = 2 - \frac{1}{p} - \frac{2}{p^k}$,

for $u_1, v_1 \in V_1$ and $v_2 \in V_2$, where $V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1},..., (p-1)p^{k-1}\}$, and $V_2 = \{0, 1, 2, 3,..., p^k - 1\} \setminus V_1$.

Proof

We consider two cases as follows.

Case 1 $u_1 v_1$ is an edge.

For edge $u_1 v_1$, by Theorem 2.2, then $\deg(u_1) = \deg(v_1) = p^k - 1$. Moreover, the vertex u_1 is adjacent to $p-2$ vertices in V_1 and is adjacent to $p^k - p$ edges in V_2. According to Theorem 9, there are $p-2$ edges with the membership value of $\frac{1}{p}$ and there are $p^k - p$ edges with the membership value of $\frac{1}{p^k}$. Analogously, for the vertex v_1, we have

$$d(u_1, v_1) = \sum_{w_1 u_1 \in E, w_1, u_1 \in V_1} \mu(w_1, u_1) + \sum_{v_1 w_2 \in E, v_1 \in V_1, w_2 \in V_2} \mu(v_1, w_2)$$

$$= 2(p-2)\cdot\mu(w_1, u_1) + 2(p^k - p)\cdot\mu(v_1, w_2)$$

$$= 2(p-2)\cdot\frac{1}{p} + 2(p^k - p)\cdot\frac{1}{p^k}$$

$$d(u_1, v_1) = 4 - \frac{4}{p} - \frac{2p}{p^k}.$$

Case 2 $u_1 v_2$ is an edge

For edge $u_1 v_2$, according to Theorem 2.2, then $\deg(v_2) = p$. Moreover, vertex u_1 is adjacent to a number $p-1$ vertices in V_1 and to $p^k - p - 2$ vertices in V_2. On the other hand, vertex v_2 is adjacent to $p-1$ vertices in V_1. Hence, by Theorem 3.2, we get

$$d(u_1, v_2) = \sum_{w_1 u_1 \in E, w_1, u_1 \in V_1} \mu(w_1, u_1) + \sum_{u_1 v_2 \in E, u_1 \in V_1, v_2 \in V_2} \mu(u_1, v_2) + \sum_{v_2 x_1 \in E, v_2 \in V_2, x_1 \in V_1} \mu(v_2, x_1)$$

$$= (p-1)\cdot\mu(w_1, u_1) + (p^k - p - 2)\cdot\mu(u_1, v_2) + (p-1)\cdot\mu(v_2, x_1)$$

$$= (p-1)\cdot\frac{1}{p} + (p^k - p - 2)\cdot\frac{1}{p^k} + (p-1)\cdot\frac{1}{p^k}$$

$$d(u_1, v_2) = 2 - \frac{1}{p} - \frac{2}{p^k}.$$

Theorems 3.5 to 3.6 discuss the general formulas for the Zagreb indices (first, second, and entire) of a fuzzy graph.

Theorem 3.5 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V\sigma, \mu)$ be the fuzzy graph of the prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = p^k$ and $k \geq 2$ are integers. Then the first Zagreb of $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$.

$$M_1\left(\widetilde{\Gamma_{\mathbb{Z}_{pk}}}\right) = \left(2 - \frac{1}{p} - \frac{p}{p^k}\right)^2 + (p-1)\cdot\left(\frac{2}{p} - \frac{1}{p^2} - \frac{1}{p^k}\right)^2 + (p^k - p)\cdot\frac{p^2}{p^{4k}}.$$

Proof

Let the vertex set of $\Gamma_{\mathbb{Z}_n}$ is partitioned into two sets,

$V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1},..., (p-1)p^{k-1}\}$, and $V_2 = \{0, 1, 2, 3,..., p^k - 1\} \setminus V_1$.

Based on the definition of the membership value of a vertex in Theorem 3.1, we get $\sigma(0) = 1.00$, $\sigma(v_1) = \frac{1}{p}$, and $\sigma(v_2) = \frac{1}{p^k}$ for every $v_1 \in V_1$ and $v_2 \in V_2$. Moreover, the number of vertices in $\Gamma_{\mathbb{Z}_n}$ is p^k where there are p vertices which have a degree of $p^k - 1$ for vertices in V_1 and there are $p^k - p$ vertices with a degree of p for vertices in V_2. It implies that

$$M_1\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \sum_{v \in V}(\sigma(v)d(v))^2 = (\sigma(0)d(0))^2 + \sum_{v_1 \in V_1}(\sigma(v_1)d(v_1))^2 + \sum_{v_2 \in V_2}(\sigma(v_2)d(v_2))^2$$

$$= \left(1\cdot\left((p-1)\cdot\frac{1}{p} + (p^k - p)\cdot\frac{1}{p^k}\right)\right)^2$$

$$+ (p-1)\left(\frac{1}{p}\cdot\left((p-1)\cdot\frac{1}{p} + (p^k - p)\cdot\frac{1}{p^k}\right)\right)^2 + (p^k - p)\left(\frac{1}{p^k}\cdot\frac{p}{p^k}\right)$$

$$M_1\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \left(2 - \frac{1}{p} - \frac{p}{p^k}\right)^2 + (p-1)\left(\frac{2}{p} - \frac{1}{p^2} - \frac{1}{p^k}\right)^2 + (p^k - p)\left(\frac{1}{p^k}\cdot\frac{p}{p^k}\right)$$

Theorem 3.6 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V\sigma, \mu)$ be the fuzzy graph of prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = p^k$ and $k \geq 2$ are integers. Then the second Zagreb of $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}$.

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{pk}}}\right) = \frac{1}{2}\cdot\frac{p-1}{p}\left(2 - \frac{1}{p} - \frac{p}{p^k}\right)^2\left(\frac{3}{2} - \frac{1}{p}\right) + \frac{1}{2}\cdot\frac{1}{p^{2k}}(p^k - p)\left(2 - \frac{1}{p} - \frac{p}{p^k}\right)(2p-1).$$

Proof

By the same manner of Theorem 3.5, we have the vertices in $\Gamma_{\mathbb{Z}_n}$ as follows:

$V_1 = \{0, p^{k-1}, 2p^{k-1}, 3p^{k-1},..., (p-1)p^{k-1}\}$,
$V_2 = \{0, 1, 2, 3,..., p^k - 1\} \setminus V_1$.

We need to consider four cases as given herewith.

Case 1 Vertex 0 is adjacent to all vertices in V_1

The number of edges connecting vertex 0 to a vertex in V_1 is $p-1$. We then obtain

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\sum_{0v_1\in E}\sigma(0)d(0)\sigma(v_1)d(v_1)$$

$$= \frac{1}{2}(p-1)\left(1\cdot\left((p-1)\cdot\frac{1}{p}+\left(p^k-p\right)\cdot\frac{1}{p^k}\right)\cdot\frac{1}{p}\cdot\left((p-1)\cdot\frac{1}{p}+\left(p^k-p\right)\cdot\frac{1}{p^k}\right)\right)$$

Case 2 Vertex 0 is adjacent to all vertices in V_2

The number of edges connecting vertex 0 to a vertex in V_2 is $p^k - p$. Therefore, we get

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\sum_{0v_2\in E}\sigma(0)d(0)\sigma(v_2)d(v_2)$$

$$= \frac{1}{2}\left(p^k-p\right)\left(1\cdot\left((p-1)\cdot\frac{1}{p}+\left(p^k-p\right)\cdot\frac{1}{p^k}\right)\cdot\frac{1}{p^k}\cdot\frac{p}{p^k}\right)$$

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\cdot\frac{1}{p^{2k}}\cdot p\left(p^k-p\right)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)$$

Case 3 Vertex in V_1 is connected to vertex in V_1

Since any pair of distinct vertices in V_1 is adjacent, there are $C(p-1, 2)$ edges. Then

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\sum_{u_1v_1\in E}\sigma(u_1)d(u_1)\sigma(v_1)d(v_1)$$

$$= \frac{1}{2}C(p-1,2)\left(\frac{1}{p}\cdot\left((p-1)\cdot\frac{1}{p}+(p^k-p)\cdot\frac{1}{p^k}\right)\cdot\frac{1}{p}\cdot\left((p-1)\cdot\frac{1}{p}+(p^k-p)\cdot\frac{1}{p^k}\right)\right)$$

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{4}\cdot\frac{1}{p^{2k}}\cdot(p-1)(p-2)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2$$

Case 4 Vertex in V_2 is adjacent to all vertices in V_1

The number of edges connecting vertex in V_2 to V_1 is $(p^k - p)(p-1)$. Hence, we obtain

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\sum_{u_2v_1\in E}\sigma(u_2)d(u_2)\sigma(v_1)d(v_1)$$

$$= \frac{1}{2}\left(p^k-p\right)(p-1)\left(\frac{1}{p^k}\cdot p^{k-1}\cdot\frac{1}{p}\left((p-1)\cdot\frac{1}{p}+\left(p^k-p\right)\cdot\frac{1}{p^k}\right)\right)$$

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\cdot\frac{1}{p^{2k}}\cdot\left(p^k-p\right)(p-1)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)$$

The summation of the results from four cases earlier implies

$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\sum_{0v_1\in E}\sigma(0)d(0)\sigma(v_1)d(v_1)+\frac{1}{2}\sum_{0v_2\in E}\sigma(0)d(0)\sigma(v_2)d(v_2)+\frac{1}{2}\sum_{u_1v_1\in E}\sigma(u_1)d(u_1)\sigma(v_1)d(v_1)+$$
$$\frac{1}{2}\sum_{u_2v_1\in E}\sigma(u_2)d(u_2)\sigma(v_1)d(v_1)$$
$$= \frac{1}{2}\cdot\frac{p-1}{p}\left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2+\frac{1}{2}\cdot\frac{1}{p^{2k}}\cdot p\left(p^k-p\right)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)+\frac{1}{4}\cdot\frac{1}{p^{2k}}\cdot(p-1)(p-2)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2+\frac{1}{2}\cdot\frac{1}{p^{2k}}\cdot$$
$$(p-1)(p-2)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2+\frac{1}{2}\cdot\frac{1}{p^{2k}}\cdot(p^k-p)(p-1)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)$$
$$M_2\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}\cdot\frac{p-1}{p}\cdot\left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2\left(\frac{3}{2}-\frac{1}{p}\right)+\frac{1}{2}\cdot\frac{1}{p^{2k}}\left(p^k-p\right)\left(2-\frac{1}{p}-\frac{p}{p^k}\right)(2p-1)$$

Theorem 3.7 Let $\widetilde{\Gamma_{\mathbb{Z}_{p^k}}} = (V\sigma, \mu)$ be the fuzzy graph of prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with $n = p^k$ and $k \geq 2$ are integers. Then the entire Zagreb of G

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2+(p-1)\left(\left(\frac{2}{p}-\frac{1}{p^2}-\frac{1}{p^k}\right)^2+\frac{1}{2}p\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2\right)+p\left(p^k-p\right)\left(\frac{p}{p^{4k}}+\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2\right).$$

Proof

There are two steps in this proof. First, we need to prove the summation of the square of the product of the membership value and the degree of the vertex, which is $\Sigma_{u\in V\left(\Gamma_{\mathbb{Z}_{p^k}}\right)}(\sigma(u)d(u))^2$. Next, it is compulsory to show the summation of the square of the product of the membership value and the degree of edge, $\Sigma_{uv\in E\left(\Gamma_{\mathbb{Z}_{p^k}}\right)}(\mu(u,v)d(u,v))^2$. However, $\Sigma_{u\in V\left(\Gamma_{\mathbb{Z}_{p^k}}\right)}(\sigma(u)d(u))^2$ is the first Zagreb index; consequently, it is sufficient to describe the second step. Now, based on the facts in Theorems 3.2 and 3.4. By the same argument of Theorem 3.5, we can state four cases as follows:

Case 1 vertex 0 is adjacent to all vertices in V_1

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \sum_{0v_1\in E}\left(\mu(0,v_1)d(0,v_1)\right)^2 = (p-1)\left(\frac{1}{p}\left(4-\frac{4}{p}-\frac{2p}{p^k}\right)\right)^2$$

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = (p-1)\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2$$

Case 2 vertex 0 is adjacent to all vertices in V_2

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \sum_{0v_2\in E}\left(\mu(0,v_2)d(0,v_2)\right)^2 = (p^k-p)\left(\frac{1}{p^k}\left(2-\frac{1}{p}-\frac{2p}{p^k}\right)\right)^2$$

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = (p^k-p)\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2$$

Case 3 Any vertex in V_1 is adjacent to all vertices in V_1

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \sum_{u_1v_1\in E}\left(\mu(u_1,v_1)d(u_1,v_1)\right)^2 = C(p-1,2)\left(\frac{1}{p}\left(4-\frac{4}{p}-\frac{2p}{p^k}\right)\right)^2$$

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \frac{1}{2}(p-1)(p-2)\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2$$

Case 4 Any vertex in V_2 is adjacent to all vertices in V_1

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = \sum_{u_2v_1\in E}\left(\mu(u_2,v_1)d(u_2,v_1)\right)^2 = (p^k-p)(p-1)\left(\frac{1}{p^k}\left(2-\frac{1}{p}-\frac{2}{p^k}\right)\right)^2$$

$$M_e\left(\widetilde{\Gamma_{\mathbb{Z}_{p^k}}}\right) = (p^k-p)(p-1)\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2$$

According to Theorem 3.5 and the four cases mentioned earlier, we obtain

$$M_e\left(\tilde{\Gamma}_{\mathbb{Z}_{p^k}}\right) = M_1\left(\tilde{\Gamma}_{\mathbb{Z}_{p^k}}\right) + \sum_{0v_1\in E}\left(\mu(0,v_1)d(0,v_1)\right)^2 + \sum_{0v_2\in E}\left(\mu(0,v_2)d(0,v_2)\right)^2$$
$$+ \sum_{u_1v_1\in E}\left(\mu(u_1,v_1)d(u_1,v_1)\right)^2 + \sum_{u_2v_1\in E}\left(\mu(u_2,v_1)d(u_2,v_1)\right)^2$$
$$= \left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2 + (p-1)\left(\frac{2}{p}-\frac{1}{p^2}-\frac{1}{p^k}\right)^2 + (p^k-p)\left(\frac{1}{p^k}\cdot\frac{p}{p^k}\right)$$
$$+ (p-1)\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2 + (p^k-p)\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2$$
$$+ \frac{1}{2}(p-1)(p-2)\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2 + (p^k-p)(p-1)\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2$$

$$M_e\left(\tilde{\Gamma}_{\mathbb{Z}_{p^k}}\right) = \left(2-\frac{1}{p}-\frac{p}{p^k}\right)^2 + (p-1)\left(\left(\frac{2}{p}-\frac{1}{p^2}-\frac{1}{p^k}\right)^2 + \frac{1}{2}p\left(\frac{4}{p}-\frac{4}{p^2}-\frac{2}{p^k}\right)^2\right) + p\left(p^k-p\right)\left(\frac{p}{p^{4k}}+\left(\frac{2}{p^k}-\frac{1}{p^{k+1}}-\frac{2}{p^{2k}}\right)^2\right)$$

Example 3.1 Consider the graph shown in Example 2.2. Let $\sigma(0) = 1$, $\sigma(8) = 0.5$, $\sigma(x) = 0.0625$, $\mu(0,8) = 0.5$ and $\mu(x, w) = 0.0625$ for $x, w \in V(\Gamma_{\mathbb{Z}_{16}})\backslash\{0, 8\}$. Subsequently, we obtain $d(0) = d(8) = 0.75$, $d(0, 8) = 1.75$ and $d(u, v) = 1.375$. To quantify the effect of the membership function on the resulting index values, we compare the obtained indices with those computed under the weighting scheme adopted in previous studies. Suppose that $\sigma(u) = 0.05$ and $\mu(u, v) = 0.05$ for every $u, v \in V(\Gamma_{\mathbb{Z}_{16}})$, then we have $d(0) = d(8)$ 0.75, $d(w) = 0.10$, $d(0, 8) = 1.4$, and $d(x, w) = 0.75$ for $x, w \in V(\Gamma_{\mathbb{Z}_{16}})\backslash\{0, 8\}$. This comparison demonstrates that the choice of membership function directly affects the resulting topological index values. In contrast to the weighting scheme adopted in previous studies, the membership function employed in this study is defined in terms of the orders of group elements via the least common multiple (LCM), so that the membership values are determined systematically by the underlying algebraic structure of the group. Therefore, by Definitions 2.2, 2.3, and 2.4 and Theorems 3.5, 3.6, and 3.7, we obtain the following table.

Topological indices	Fuzzy graph	Fuzzy prime coprime graph
first Zagreb	0.0032	2.3641
second Zagreb	0.1061	0.0027
entire Zagreb	3.3365	0.0474

From a computational perspective, the evaluation of Zagreb indices on a fuzzy graph fundamentally depends on the cardinalities of the vertex and edge sets together with the prescribed membership functions (Kalathian et al. 2020). For a fuzzy prime coprime graph $\Gamma_{\mathbb{Z}_n}$ with n vertices, adjacency is determined by examining each unordered pair of distinct vertices u, v and verifying whether $\gcd(|u|, |v|) = 1$ or $\gcd(|u|, |v|)$ is prime. Hence, a direct construction requires at most $O(n^2)$ pairwise adjacency checks. The corresponding membership values are subsequently determined from the orders of the group elements and their least common multiples. Although this construction involves additional arithmetic operations compared with fuzzy graphs whose edge structure is predetermined, it provides a weighting scheme derived explicitly from the underlying algebraic structure of the group. This distinguishes the proposed approach from the fuzzy pizza graph, in which prescribed membership values are used to construct the Zagreb indices (Mufti et al. 2023a). Consequently, the resulting Zagreb indices depend not only on the graph structure but also encode information arising from the order structure of the underlying group.

CONCLUSION

This study presents a mathematical framework for analyzing the fuzzy prime coprime graph of integer modulo groups, in which vertex and edge memberships are defined systematically based on the greatest common divisor (gcd) and least common multiple (lcm) of the orders of group elements. The main results include the classification of membership values for vertices and edges, as well as the derivation of closed-form expressions for the first Zagreb, second Zagreb, and entire Zagreb indices of the fuzzy graph. These results deepen the understanding of how fuzzy set theory can be integrated with algebraic graph structuresand provide a solid foundation for further theoretical and computational investigations.

A limitation of this study is that the analysis is restricted to prime-power orders, so the derived formulas may not directly apply to more general composite-order cases. Future research may extend the construction to composite orders, other classes of finite groups or algebraic structures, and additional degree-based, distance-based, or spectral topological indices. These extensions may provide a broader understanding of the relationship between fuzzy graph invariants and algebraic structures.

ACKNOWLEDGMENTS

We would like to express our sincere gratitude to the Faculty of Mathematics and Natural Sciences, University of Mataram, for the research funding provided through the 2025 PNBP Grant under the Beginner Lecturer Research (PDP) scheme. This support was instrumental in the completion of this study.

REFERENCES

Abdurahim, Romdhini, M.U. & Husain, S.K.S. 2025. Zagreb-based indices of prime coprime graph for integers modulo power of primes. *Malaysian Journal of Mathematical Sciences* 19(3): 961-974.

Abdurahim, Romdhini, M.U., Al-Sharqi, F. & Robbaniyyah, N.A. 2025. Relative prime coprime graph of integers modulo group and its reverse topological indices. *Pan American Journal of Mathematics* 4(10): 1-9.

Abdurahim, Pratiwi, L.F., Karang, G.Y., Wardhana, I.G.A.W., Irwansyah, Awanis, Z.Y. & Romdhini, M.U. 2026. Indeks Wiener pada graf koprima prima dari grup bilangan bulat modulo. *Jurnal Matematika UNAND* 15(2): 249-258.

Adhikari, A. & Banerjee, S. 2022. Prime coprime graph of a finite group. *Novi Sad Journal of Mathematics* 52(2): 41-59.

Afdhaluzzikri, M., Wardhana, I.G.A.W., Maulana, F. & Biswas, H.R. 2025. The non-coprime graphs of upper unitriangular matrix groups over the ring of integer modulo with prime order and their topological indices. *BAREKENG: Jurnal Ilmu Matematika dan Terapan* 19(1): 547-556.

Al-Mutab, H.M. 2019. Fuzzy graphs. *Journal of Advances in Mathematics* 17: 232-247.

Fayazi, F., Mahmudi, F. & Gholami, A. 2020. Some properties of fuzzy star graph and fuzzy line graph. *Jordan Journal of Mathematics and Statistics* 3(1): 139-151.

Hao, S., Zhong, G. & Ma, X. 2022. Notes on the co-prime order graph of a group. *Proceedings of the Bulgarian Academy of Sciences* 75(3): 340-348.

Islam, S.R. & Pal, M. 2023. Second Zagreb index for fuzzy graphs and its application in mathematical chemistry. *Iranian Journal of Fuzzy Systems* 20(1): 119-136.

Islam, S.R. & Pal, M. 2021. First Zagreb index on a fuzzy graph and its application. *Journal of Intelligent & Fuzzy Systems* 40(6): 10575-10587.

Jana, U. & Ghorai, G. 2023. First entire Zagreb index of fuzzy graph and its application. *Axioms* 12(5): 415.

Kalathian, S., Ramalingam, S., Raman, S. & Srinivasan, N. 2020. Some topological indices in fuzzy graphs. *Journal of Intelligent and Fuzzy Systems* 39(5): 6033-6046.

Mufti, Z.S., Tabraiz, A., Xin, Q., Almutairi, B. & Anjum, R. 2023a. Fuzzy topological analysis of pizza graph. *AIMS Mathematics* 8(6): 12841-12856.

Mufti, Z.S., Anjum, R., Tawfiq, F.M.O., Tabraiz, A., Xin, Q. & Akter, D. 2023b. Fuzzy topological characterization of qC_n graph via fuzzy topological indices. *Journal of Mathematics* 2023: 5141179.

Nurhabibah, N., Wardhana, I.G.A.W. & Switrayni, N.W. 2023. Numerical invariants of coprime graph of a generalized quaternion group. *Journal of the Indonesian Mathematical Society* 29(1): 36-44.

Romdhini, M.U., Abdurahim, Maharani, A.E.S.H. & Kamali, S.R. 2025a. Transmission-based energies of prime coprime graph for integers modulo group. *Science and Technology Indonesia* 10(3): 759-765.

Romdhini, M.U., Abdurahim, Al-Sharqi, F., Bahri, S. & Maharani, A.E.S.H. 2025b. On energy of prime coprime graph for dihedral groups. *Mathematics and Statistics* 13(3): 175-179.

Romdhini, M.U., Al-Sharqi, F., Nawawi, A., Al-Quran, A. & Rashmanlou, H. 2023. Signless Laplacian energy of interval-valued fuzzy graph and its applications. *Sains Malaysiana* 52(07): 2127-2138.

Saini, M., Kumari, V., Rana, P., Sehgal, A. & Singh, D. 2025. Degree based matrices of co-prime order graph of finite groups. *Mathematics and Statistics* 13(3): 127-135.

Sehgal, A., Manjeet & Singh, D. 2021. Co-prime order graphs of finite Abelian groups and dihedral groups. *Journal of Mathematics and Computer Science* 23: 196-202.

Tabraiz, A., Mufti, Z.S., Aslam, M.N., Saleem, N. & Zadeh, H. 2023. Fuzzy computational analysis of flower graph via fuzzy topological indices. *Journal of Mathematics* 2023: 8077729.

Wang, Y., Shahzadi, A., Binyamin, M.A., Mehtab, M., Tchier, F. & Aslam, A. 2024. On bounds for the atom bond sum connectivity index of graphs associated with symmetric numerical semigroups. *AKCE International Journal of Graphs and Combinatorics* 22(2): 125-135.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control* 8: 338-353.