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Stability of Cross-Gradient Joint Inversion under Noisy Conditions: A Systematic Review

(Kestabilan Penyongsangan Bersama Kecerunan Silang di bawah Keadaan Hingar: Satu Kajian Sistematik)

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ABSTRACT

Cross-gradient joint inversion (CGJI) is a widely applied geophysical method that integrates multiple parameters by enforcing structural similarity through cross-gradient constraints. While its effectiveness has been demonstrated across diverse geological settings, its stability under noisy conditions remains insufficiently characterised. This study presents a systematic literature review (SLR) of 68 publications between 2015 and 2024, following a structured identification, screening, and synthesis protocol based on PRISMA guidelines. The reviewed studies were analysed in terms of applications, methodological developments, and noise stability using a semi-quantitative synthesis approach. The results indicate that CGJI is most commonly applied in geodynamics–tectonics and petroleum exploration, with increasing methodological developments driven by advances in machine learning. Its performance is strongly dependent on the integration of datasets with complementary sensitivities, rather than on any specific geophysical method. Most studies report improved structural delineation and reduced inversion ambiguity compared to single-method approaches, particularly under low-to-moderate noise conditions (≤ 20%). However, a critical gap remains: existing evaluations are predominantly based on synthetic datasets, and noise levels rarely exceed 20%, which may not reflect realistic field conditions. Although metrics such as Root Mean Square (RMS) and Structural Similarity Index Measure (SSIM) are used to assess noise sensitivity, their application remains limited and inconsistent. These findings highlight the need for systematic evaluation of CGJI under higher noise levels and real field conditions to ensure its reliability and practical applicability.

Keywords: Cross gradient joint inversion; geophysical applications; methodological development; noise stability; systematic literature review

ABSTRAK

Penyongsangan bersama kecerunan silang (CGJI) ialah kaedah geofizik yang digunakan secara meluas yang mengintegrasikan pelbagai parameter dengan menguatkuasakan keserupaan struktur melalui kekangan kecerunan silang. Walaupun keberkesanannya telah ditunjukkan merentasi pelbagai persekitaran geologi, kestabilannya di bawah keadaan hingar masih belum dicirikan dengan mencukupi. Penyelidikan ini membentangkan satu tinjauan kepustakaan sistematik (SLR) terhadap 68 penerbitan antara tahun 2015 hingga 2024, mengikut protokol pengenalpastian, penapisan dan sintesis yang berstruktur berdasarkan garis panduan PRISMA. Kajian yang dikaji dianalisis dari segi aplikasi, perkembangan metodologi dan kestabilan hingar menggunakan pendekatan sintesis separa kuantitatif. Keputusan menunjukkan bahawa CGJI paling kerap digunakan dalam geodinamik-tektonik dan penerokaan petroleum dengan peningkatan perkembangan metodologi didorong oleh kemajuan dalam pembelajaran mesin. Prestasinya sangat bergantung pada penyepaduan set data dengan kepekaan pelengkap dan bukannya pada mana-mana kaedah geofizik tertentu. Kebanyakan kajian melaporkan peningkatan dalam penentuan struktur dan pengurangan ketaksaan penyongsangan berbanding pendekatan kaedah tunggal, terutamanya di bawah keadaan hingar rendah hingga sederhana (≤ 20%). Walau bagaimanapun, terdapat jurang kritikal masih wujud, penilaian sedia ada kebanyakannya berdasarkan data sintetik dan tahap hingar jarang melebihi 20% yang mungkin tidak mencerminkan keadaan lapangan sebenar. Walaupun metriks seperti ralat Punca Purata Kuasa Dua (RMS) dan Ukuran Indeks Kesamaan (SSIM) digunakan untuk menilai kepekaan hingar, penggunaannya masih terhad dan tidak tekal. Keputusan ini menekankan keperluan untuk penilaian sistematik terhada CGJI di bawah tahap hingar yang lebih tinggi dan keadaan lapangan sebenar bagi memastikan kebolehpercayaan dan kebolehgunaan praktikalnya.

Kata kunci: Aplikasi geofizik; kestabilan hingar; penyongsangan bersama kecerunan silang; perkembangan metodologi; tinjauan kepustakaan sistematik

INTRODUCTION

Cross-gradient joint inversion (CGJI) is a technique that integrates two or more geophysical parameters by applying mathematical operations to a vector known as the cross-gradient. This approach is widely used to reduce ambiguities in geophysical model interpretation, since observational data of physical parameters often permit multiple plausible models (Wang, Tan & Wang 2017). Moreover, CGJI helps mitigate issues of subsurface misinterpretation that arise from the non-linearity of geophysical inverse problems and the inherent resolution limitations of individual methods (Zakaria et al. 2024, 2022). By combining complementary datasets, CGJI provides more reliable structural information, as each parameter describes a distinct aspect of the Earth's physical properties (Zakaria et al. 2025).

CGJI enforces structural similarity between independent datasets (for example gravity, magnetics, resistivity and seismic velocity) by penalising the cross-product of their gradients. This constraint tends to align subsurface features across models, producing results that are more geologically coherent and less ambiguous than standalone inversions (Giraud et al. 2021; Zhu & Harris 2015). The method has been applied in a variety of contexts, including mineral exploration (Meng et al. 2022), hydrocarbon prospecting (Meju et al. 2023), groundwater assessment (von Ketelhodt et al. 2019) and tectonic studies (Wu et al. 2020), demonstrating its versatility for multi-parameter imaging.

Nevertheless, CGJI presents challenges. A principal concern is its sensitivity to noise arising from measurement errors, site conditions and processing artefacts (Gross 2019). Many studies that evaluate CGJI limit tests to synthetic models with relatively low noise levels, so the method's stability in high-noise, real-world settings remains inadequately characterised (Zhang et al. 2019b; Zhu et al. 2023). This gap raises practical concerns about the reliability of CGJI for routine field applications.

Despite the growing body of literature on joint inversion, existing reviews have predominantly focused on the mathematical frameworks or broad geophysical applications. For instance, Gallardo and Meju (2003) provided a foundational overview of physical property relationships, while Moorkamp (2017) synthesized the progress of multi-physics integration and algorithmic advancements. However, these reviews often evaluate performance under relatively 'clean' synthetic scenarios or low-noise field data. There remains a critical void in systematic evaluations regarding the 'noise-breakdown point' of cross-gradient constraints. To date, no systematic literature review (SLR) has specifically quantified how CGJI behaves as noise levels approach, a threshold where structural coupling often becomes unstable. This study fills this gap by utilising the PRISMA framework to critically assess the stability of CGJI across a decade of published research. By identifying trends, limitations and methodological advances, we aim to provide a consolidated perspective and practical guidance for future research.

THEORETICAL OVERVIEW

Before discussing the application and development of CGJI, it is important to understand its theoretical foundations and the principles governing its operation. Inversion in geophysics refers to a suite of mathematical and statistical techniques designed to reconstruct subsurface properties from measured observational data (Gosselin et al. 2022). Traditionally, when two or more geophysical methods are employed, each dataset is inverted independently, often leading to models that are internally consistent but structurally incompatible. The advent of joint inversion strategies has significantly improved this situation, offering more coherent reconstructions and models that provide a better fit to physical reality (Song et al. 2019; Wang, Tan & Wang 2017).

The cross-gradient method (Gallardo & Meju 2003) provides a practical way to couple independent property models without assuming an explicit petrophysical relation. For two geophysical parameters, $\boldsymbol{p}$ and $\boldsymbol{q}$, the cross-gradient vector is expressed as (Abdullah Sanusi et al. 2021; Gallardo & Meju 2003; Qiao et al. 2024; Shi et al. 2017; Zhao et al. 2023):

$$t(x, y, z) = \nabla \boldsymbol{p}(x, y, z) \times \nabla \boldsymbol{q}\,(x, y, z) \tag{1}$$

with $\boldsymbol{t}$ is the cross gradient of the parameters p and q. If $\boldsymbol{t} = 0$, the gradients of $\boldsymbol{p}$ and $\boldsymbol{q}$ are parallel, indicating that both parameters exhibit coincident structural features. In this case, the models reinforce each other, providing a stable constraint in the inversion. Conversely, a non-zero cross-gradient indicates structural inconsistency, which destabilises the inversion process by making the objective function non-unique (Molodtsov, Kiyan & Bean 2024).

In practice, adding the cross-gradient constraint often improves structural resolution, reduces ambiguity and yields models that better reflect geological expectations compared with independent inversions (Giraud et al. 2021; Meju et al. 2019). CGJI has therefore been effective in delineating interfaces such as lithological boundaries, fault zones and aquifer contacts (Meng et al. 2022; Zhou et al. 2015).

However, the same coupling mechanism creates vulnerabilities. Because the constraint operates on spatial gradients, it is fundamentally agnostic to the origin of gradient differences: genuine geological contrasts and noise-induced fluctuations are treated in the same mathematical manner. Consequently, noise present in one dataset can be propagated, and in some circumstances amplified, into the coupled model (Molodtsov, Kiyan & Bean 2024). The extent of such propagation depends on factors including relative parameter sensitivities, the weighting applied to the cross-gradient term, and the chosen regularisation strategy (Al Nasser & Morgan 2021; Zhang et al. 2019b). If one dataset is markedly noisier or less sensitive, it may dominate the joint solution and thereby negate the intended benefit of data integration.

Additional theoretical issues relate to prior constraints and model discretisation. Strong regularisation can suppress noise but at the cost of oversmoothing key geological features, whereas weak regularisation may leave the inversion vulnerable to noise-driven artefacts (Qin, Bohlen & Pan 2024). Iterative convergence behaviour likewise matters: excessively rapid convergence can lock in unstable features, while slower, better-controlled optimisation can improve stability albeit with higher computational cost (Zhang & Wang 2019).

Taken together, these theoretical considerations explain why noise stability is a critical frontier for CGJI. Rigorous, systematic evaluation under realistic noise conditions is necessary to determine whether improved structural alignment translates into reliable subsurface models in practice.

DATA AND METHODS

This research uses a systematic literature review (SLR) approach tailored to assess how CGJI has been applied and evaluated with respect to noise stability. The review comprises three main stages: identification, screening and bibliographical analysis.

IDENTIFICATION

Records were retrieved using the Publish or Perish application (Harzing 2007) from the Scopus and Web of Science (WoS) databases for the period 2015-2024. Search strings combined terms such as 'cross-gradient joint inversion', 'joint inversion noise stability', and 'CGJI applications'. Only peer-reviewed, English-language publications were considered. Foundational studies predating 2015 (Gallardo & Meju 2003) are cited in the Introduction and Theoretical Overview section to provide theoretical context, but the systematic screening of literature was restricted to publications from 2015-2024.

SCREENING

A total of 211 records were initially retrieved from Scopus and Web of Science (WoS). After removing duplicates (n = 23), the remaining records were screened based on titles and abstracts. Non-English publications were excluded (n = 18), followed by the removal of documents that were not accessible (n = 11) such as conference abstracts without full papers.

Only studies within the geophysical domain were retained (n = 141). Screening was conducted by the first author, with ambiguous cases cross-checked against predefined criteria. Studies were included if they explicitly applied or developed CGJI (or structurally coupled joint inversion) in synthetic and/or field contexts. Exclusion criteria comprised non-peer-reviewed documents and theoretical studies that did not address CGJI applications (n = 73). This screening process resulted in a final dataset of 68 studies. The overall workflow is summarised in a PRISMA flow diagram (Figure 1). This structured protocol supports a systematic and analytical synthesis of how noise has been addressed in CGJI literature.

To ensure the scientific rigour of this review, a quality assessment was conducted on the selected documents. The dataset is predominantly composed of high-impact peer-reviewed literature, with approximately 85% of the papers published in journals ranked as Q1 or Q2 journals according to the Scimago Journal Rank (SJR) or Web of Science (WoS) classifications, including *Geophysics*, *Journal of Geophysical Research: Solid Earth*, and *Geophysical Journal International*. This ensures that the reported findings on noise behaviour and inversion stability are grounded in well-established and rigorously reviewed research.

BIBLIOGRAPHICAL ANALYSIS AND SYNTHESIS

VOSviewer application (https://app.vosviewer.com/) was employed to establish bibliometric networks based on the filtered data (Amin et al. 2024). These networks were subsequently used to support the identification of dominant research themes and their evolution over time. This tool enables the visualisation of research topics and thematic linkages. Using VOSviewer, relevant information such as trending topics and potential directions for future work in CGJI was extracted. Particular attention was paid to the introduction, research methods, and discussion sections of the selected papers.

Following the screening process, a total of N = 68 studies were retained for detailed analysis. This dataset forms the basis for all subsequent bibliographical and synthesis analyses. Eligible studies were grouped thematically into: (i) applications (hydrocarbon, mineral, groundwater, tectonics), (ii) methodological development (computational acceleration, hybrid parameterisation, machine learning integration), and (iii) noise stability assessment. Each study was classified based on its primary research focus, although some studies contribute to multiple categories. Within each group, papers were critically evaluated for how they quantified and treated noise, the testing strategy employed (synthetic perturbation, controlled field tests, or real field data), performance metrics reported Root Mean Square (RMS) perturbation, and Structural Similarity Index Measure (SSIM), and the extent of independent validation (for example borehole data). These thematic categories are not mutually exclusive.

In addition to thematic classification, several key methodological aspects were also analysed to identify patterns in the literature. Studies were classified according to the type of data used, including synthetic data, field data, or a combination of both. Of the total 68 studies, approximately 63% used a combination of synthetic and field data, while 13% used only synthetic data and 24% used only field data.

In terms of model performance evaluation, the majority of studies used Root Mean Square (RMS) error as the primary indicator, with around 68 studies (70.6%) adopting this parameter, while only 3 studies (5%) used the Structural Similarity Index Measure (SSIM), with the remainder using other statistical parameters (17%) such as standard deviation. These evaluation metrics are not mutually exclusive, as some studies employ more than one metric for validation.

Furthermore, the geophysical parameters used in the CGJI studies were analysed in aggregate to identify the dominance of data types. The results show that potential field-based parameters such as gravity and magnetics were the most frequently used in approximately 41 studies (60%), followed by electrical parameters such as resistivity in approximately 39 studies (57%), and seismic methods in approximately 27 studies (40%). As CGJI inherently involves more than one geophysical parameter, the analysis at this stage focused on the types of parameters used, while further analysis regarding parameter combinations is discussed in the following section.

The analysis in this study was conducted using a semi-quantitative synthesis approach, including the identification of recurring methodological patterns, dominant evaluation strategies, and underexplored aspects such as noise stability. Each study was analysed in depth to identify key findings, the methodologies employed, and how noise was handled in the inversion process. Subsequently, these studies were compared and linked to one another within the same categories to show common patterns, differences in approach, and trends in the reported results. Furthermore, frequency analysis was used to quantify the distribution of research focus and methodological choices across the dataset. This approach enabled the compilation of a review that is not only descriptive, but also analytical and comparative.

SYNTHESIS OF REVIEWED STUDIES

GENERAL TRENDS

The interesting results of the filtering process (Figure 1) show that the term ‘cross-gradient’ appears across various disciplines, including engineering, hydrology and the social sciences. Of the entire corpus, only 141 documents were directly related to geophysical applications, and not all of these specifically focused on cross-gradient joint inversion (CGJI) such as hydraulic tomography (Zhao et al. 2023). This indicates that although the cross-gradient concept is widely recognised, its application in geophysical inversion remains relatively limited and specialised.

As summarised in Table 1, the distribution of publications shows a general upward trend, with a peak in publications occurring around 2018 (nine publications) and activity continuing through to 2024 (five publications). This pattern reflects a sustained interest in CGJI, although the growth rate is not particularly rapid, indicating that the field is still in its developmental stage.

In terms of research focus, the literature is dominated by application-based studies and methodological development. Around 66% of studies have a primary focus on applications, while 35% emphasise method development. Furthermore, around 38% of studies explicitly mentioned the use of noise in their research, although in some cases there was overlap between categories. In studies that used noise, it was more often treated as part of the model validation process than as a primary research variable. This pattern suggests an implicit assumption within the literature that CGJI performs well under certain noise conditions, although this assumption is rarely tested systematically.

Topic network analysis (Figure 2) also shows a shift in research focus over time. During the 2015-2019 period, the main theme was dominated by structural similarity, relating to comparisons between joint inversion and separate inversion. Meanwhile, during the 2020-2024 period, the focus shifted to the explicit application of cross-gradient constraints within the joint inversion framework, with gravity and magnetic data becoming the most frequently used parameters. This shift indicates a transition from the conceptual validation stage towards broader application.

Interestingly, the term ‘accuracy’ has also begun to emerge in the most recent period. Although terms such as ‘noise’ or ‘noise stability’ are still rarely mentioned explicitly, the increasing use of terms related to accuracy may reflect growing attention to data quality. However, this interpretation must be treated with caution, as the relationship between accuracy and noise is often implicit and not analysed directly. Overall, this pattern indicates a trend towards more comprehensive performance evaluation, which has the potential to develop into a more explicit examination of noise stability in the future.

GEOPHYSICAL APPLICATIONS

The reviewed literature indicates that CGJI has been applied across various fields of geophysics, with 45 studies specifically focusing on its applications. These applications are primarily concentrated in geodynamics–tectonics and petroleum exploration, with additional contributions to mineral exploration, groundwater studies, and near-surface investigations (Table 2). This distribution reflects the suitability of CGJI for problems requiring structural consistency across different physical parameters.

The patterns observed in the selection of geophysical methods indicate that potential-field-based methods (gravity and magnetic) are frequently used in various combinations, appearing in approximately 35% of studies, and are not limited to gravity-magnetic pairs alone. Some studies combine gravity with gravity gradient or aerogravity data, while magnetic data is also frequently combined with other parameters such as resistivity or seismic methods. This variety of combinations reflects the flexibility of CGJI in integrating datasets with different sensitivities. This also indicates that no single combination dominates, reinforcing that method selection is largely problem-dependent. This variability also suggests that the effectiveness of CGJI is strongly influenced by the degree of contrast between the integrated parameters.

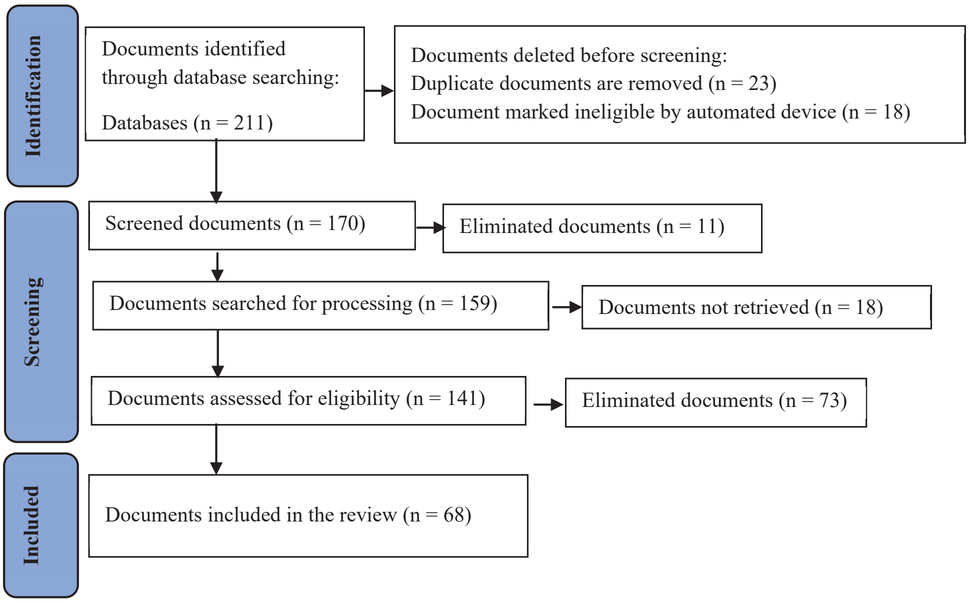

FIGURE 1. The filtering process of documents

TABLE 1. Document distribution by year

Year	Documents
2015	4
2016	2
2017	5
2018	9
2019	9
2020	6
2021	5
2022	11
2023	12
2024	5
Total	68

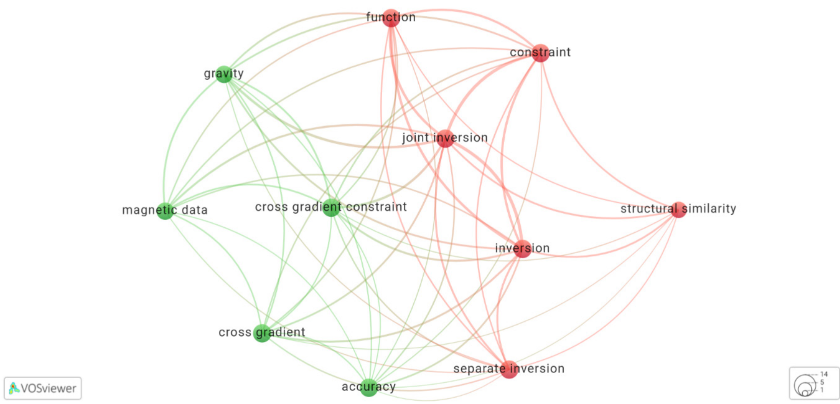

FIGURE 2. The topics network-based on bibliographical analysis

TABLE 2. Summary of applications of CGJI based on the documents

No	Application field	Parameters	No. of Studies	References
1	Petroleum/ hydrocarbon exploration	CSEM (resistivity), MT (resistivity), Resistivity, P-, & S-wave velocity	5	(Zhu & Harris 2015) (Lan et al. 2018) (Meju et al. 2019) (Saleh et al. 2022) (Saleh et al. 2023)
2	Archeology	Magnetic (susceptibility), Resistivity, & P-wave velocity	2	(Shi et al. 2017) (Varfinezhad, Oskooi & Fedi 2020)
3	Groundwater investigation	Gravity (density), Resistivity, P-, S-wave velocity, & Induced Polarization (Chargeability)	4	(von Ketelhodt et al. 2019) (Yari et al. 2021) (Zhu et al. 2023) (Ghari et al. 2024)
4	Salt intrusion	Magnetotelluric (resistivity), Gravity (density), & P-wave velocity	4	(Demirci et al. 2017) (Colombo, Rovetta & Turkoglu 2018) (Turkoglu et al. 2018) (Qiao et al. 2024)
5	Near surface	Magnetotelluric (resistivity), Induced Polarization (Chargeability), Resistivity, GPR (resistivity) & P-wave velocity	4	(Jordi et al. 2020) (Abdullah Sanusi et al. 2021) (Li, Li & Zhang 2022) (de Peppo, Cercato & De Donno 2024)
6	Mineral exploration	Magnetic (susceptibility), Gravity (density), Magnetotelluric (resistivity), P-wave velocity	9	(Zhou et al. 2015) (León-Sánchez, Gallardo & Ley-Cooper 2018) (Feng et al. 2018) (Zhang & Li 2019) (Ma et al. 2021) (Tavakoli et al. 2021) (Meng et al. 2022) (Vatankhah et al. 2022) (Wei, Li & Sun 2023)
7	Geodynamics and tectonics	Gravity (density), Magnetic (susceptibility), Magnetotelluric (resistivity), P-, & S-wave velocity,	10	(Gessner et al. 2016) (Demirci, Dikmen & Candansayar 2018) (Peng, Tan & Moorkamp 2019) (Xu et al. 2019) (Wu et al. 2020) (Martin et al. 2021) (Wu et al. 2022) (Wang et al. 2022) (Huang et al. 2023) (Meju et al. 2023)
8	Hidden low-velocity layer	Resistivity & P-wave velocity	1	(Yari et al. 2021)
9	Geothermal	Gravity (density), Magnetic (susceptibility), Magnetotelluric (resistivity), & Seismic (P-wave & surface wave)	6	(Carrillo & Gallardo 2018) (Oliver-Ocaño et al. 2019) (Yadav & Sircar 2019) (Zhao et al. 2020) (Liao et al. 2022) (Ragueh et al. 2024)

In contrast, electrical methods (including resistivity, magnetotelluric, and their derivatives) and seismic methods are more commonly used in near-surface applications, where high spatial resolution and sensitivity to lithological contrast are key factors (Ma et al. 2024). Overall, the selection of method combinations within CGJI is determined not only by the type of data available, but also by the scale of the investigation and the physical characteristics of the subsurface target. These observations indicate that CGJI does not exhibit universal superiority for a specific geophysical method; instead, its effectiveness depends on the integration of datasets with complementary sensitivities.

In particular, the combination of different methods provides complementary insights. Resistivity data are effective in identifying electrical contrasts associated with fluid content and lithology (Sharom et al. 2025; Zaid et al. 2023), while seismic data provide detailed information on stratification and mechanical properties (Ma et al. 2024). Potential field data, although generally characterised by lower resolution, play a role in characterising large-scale structures. The integration of these various datasets via CGJI enables a more balanced reconstruction of subsurface models by leveraging the strengths of each method.

Across various application domains, CGJI generally improves the delineation of structural boundaries and reduces inversion ambiguity compared to single-method approaches, particularly when the integrated datasets exhibit complementary sensitivities. However, the extent of this improvement is not uniform and depends heavily on data quality, parameter sensitivity, and the compatibility of the integrated datasets. For example, studies on groundwater (von Ketelhodt et al. 2019; Yari et al. 2021; Zhu et al. 2023) and mineral exploration (Ma et al. 2021; Tavakoli et al. 2021; Wei, Li & Sun 2023) demonstrate improvements in target identification, but these improvements tend to be more significant when the datasets used have complementary sensitivities rather than redundant ones.

Nevertheless, some limitations remain. The performance of CGJI is highly sensitive to data quality, weighting strategies, and the relative contributions of each parameter in the inversion process. Furthermore, the majority of application studies have been conducted under low to moderate noise conditions, which do not fully represent complex field conditions. Consequently, the stability of CGJI under high-noise conditions has not yet been adequately characterised, necessitating a more systematic investigation into its stability under noise.

METHODOLOGICAL DEVELOPMENT

Compared with studies focusing on applications, the number of studies explicitly addressing the methodological development in CGJI remains relatively limited (Table 3). Nevertheless, recent research indicates a clear shift towards more advanced and adaptive inversion strategies (Wang et al. 2024; Zhou et al. 2015), driven by the need to address key limitations such as computational cost, resolution, and noise sensitivity.

One of the key areas of development in CGJI is the improvement of computational efficiency and scalability. Initial efforts focused on the transition from a 2D approach to full 3D inversion, enabling the representation of more complex subsurface structures (Fregoso, Gallardo & García-Abdeslem 2015; Zhou et al. 2015). While this approach enhances the realism of the models, it results in a significant increase in computational load. Subsequent developments have addressed these limitations through algorithm optimisation and the use of parallel computing, thereby enabling the processing of large-scale datasets with reduced computation times without compromising structural consistency (Joulidehsar, Moradzadeh & Ardejani 2018; Liu et al. 2023b, 2023a).

In parallel, methodological refinements have also been directed towards improving the stability of the inversion and parameter handling. Approaches such as adaptive constraint generation, hybrid parameterisation, and stochastic frameworks have been developed to enhance stability, particularly under noisy conditions (Tarits et al. 2015; Zhang et al. 2019a). Advanced regularisation techniques and Bayesian approaches provide a more systematic mechanism for balancing data misfit and model smoothness, thereby reducing artefacts caused by noise without compromising key structural features (Fregoso, Palafox & Moreles 2020; Zhang et al. 2019b).

The latest methodological developments in CGJI involve integration with machine learning and deep learning. These data-driven approaches have the capability to model complex non-linear relationships between geophysical parameters and to accelerate the inversion process (Hu et al. 2024, 2023; Ma et al. 2024). Initial results suggest that these approaches have the potential to improve structural resolution and noise stability. However, its effectiveness remains highly dependent on the quality of the training data and the model's generalisation ability, which have not yet been fully validated under various geological conditions. This suggests that, despite its promise, machine learning-based CGJI remains in an exploratory stage rather than a fully established framework. In addition, most validations remain limited to synthetic datasets, which restricts the assessment of their performance under realistic field conditions.

Overall, the methodological development in CGJI can be understood as an effort to address three main challenges: computational efficiency, inversion stability and the complexity of data integration. Although significant progress has been made in all three aspects, several limitations remain, particularly in ensuring stable performance under high-noise conditions that more accurately represent field conditions. This highlights the need for further research that explicitly integrates methodological innovations with a systematic evaluation of the effects of noise.

TABLE 3. Summary of development of CGJI based on the documents

No	Development key	Parameters	No. of studies	References
1	3D CGJI	Gravity (density), Magnetic (susceptibility), Magnetotelluric (resistivity), P- & S-wave velocity	4	(Zhou et al. 2015) (Fregoso, Gallardo & García-Abdeslem 2015) (Geng, Yang & Huang 2017) (Huang et al. 2023)
2	Fast 3D CGJI	Gravity (density) & Magnetic (susceptibility)	3	(Joulidehsar, Moradzadeh & Ardejani 2018) (Liu et al. 2023a) (Liu et al. 2023b)
3	Stochastic correlation and automatic constraints	Gravity (density), Magnetotelluric (resistivity), Resistivity, & P-wave velocity	2	(Tarits et al. 2015) (Rittgers et al. 2016)
4	multiple and Hybrid Parameter CGJI	Gravity (density), Magnetic (susceptibility), Magnetotelluric (resistivity) & Velocity	2	(Pak, Li & Kim 2017) (Zhang et al. 2019a)
5	Advanced Regularisation and Bayesian Approaches	Gravity (density), Magnetic (susceptibility) & Magnetotelluric (resistivity)	2	(Zhang et al. 2019b) (Fregoso, Palafox & Moreles 2020)
6	Computational and Algorithmic Innovations	Gravity (density), Magnetic (susceptibility), Resistivity, Velocity, & Induced Polarization (Chargeability)	9	(Sun & Wang 2020) (Zhong et al. 2022) (He, Li & Zhang 2022) (Yang et al. 2023) (Gao & Zhang 2018) (Fang et al. 2022) (Tavakoli et al. 2021) (Ma et al. 2022) (Li et al. 2023) (Niu et al. 2023) (Vatankhah et al. 2023)
7	Deep learning	Gravity (density), Magnetic (susceptibility), Resistivity, & P-wave velocity	2	(Hu et al. 2023) (Hu et al. 2024)

NOISE STABILITY

Noise stability is one of the main criteria for evaluating the performance of cross-gradient joint inversion (CGJI). Theoretically, joint inversion is expected to be more sensitive to noise than individual inversion due to the interdependence of parameters (Wang, Tan & Wang 2017). However, empirical results from various studies indicate that the behaviour of CGJI with respect to noise is more complex than this assumption suggests and is highly dependent on the inversion design and the characteristics of the data used, as summarised in Table 4. All studies summarised in Table 4 incorporate controlled noise into synthetic datasets. This highlights that current evaluations of CGJI noise stability are largely based on synthetic scenarios, with limited validation using real field data.

One key finding is that the cross-gradient constraint, while effective in maintaining structural consistency across models, can also act as a pathway for noise propagation between parameters. In other words, errors in one dataset can influence the parameter models of another through structural coupling (Varfinezhad, Oskooi & Fedi 2020). This effect is not inherently negative, as under certain conditions it can actually help stabilise the inversion results. Therefore, noise stability in CGJI is not an intrinsic property, but rather a result dependent on several factors, such as the relative sensitivity of geophysical parameters, data weighting schemes, regularisation strategies, and model discretisation (Al Nasser & Morgan 2021; Molodtsov, Kiyan & Bean 2024; Zhang et al. 2019b). Specifically, the selection of the cross-gradient value as weighting factor in CGJI objective function plays a decisive role; an inappropriately high value in high-noise environments can lead to ‘noise leakage,’ where artifacts from a noisy dataset are erroneously forced into a cleaner dataset. Our review indicates that most current studies still rely on empirical trial-and-error for weighting, highlighting a critical need for automated, noise-adaptive weighting schemes.

Based on the reviewed literature, the majority of studies indicate that CGJI is capable of delivering better performance than individual inversion under low to moderate noise conditions (≤ 20%). This threshold reflects the most commonly tested noise range reported in the reviewed studies rather than a strict methodological limit. For example, several studies demonstrate that CGJI can successfully image subsurface structures under low noise conditions, illustrating enhanced structural recovery (Shi et al. 2017). This improvement is generally observed in the ability to maintain structural continuity and reduce the ambiguity of inversion results. Performance is typically evaluated using statistical metrics such as Root Mean Square (RMS) perturbation, which quantifies the deviation between inverted and reference models. Lower RMS values indicate better model fidelity (Zhu et al. 2023).

However, this advantage is not always consistent under more complex conditions. At higher noise levels or when there is an imbalance in sensitivity between parameters, the performance of CGJI may degrade significantly. Under these conditions, the propagation of noise between models can actually worsen the quality of the inversion results, particularly at greater depths or in areas with limited data. This demonstrates that the success of CGJI is highly dependent on the suitability and quality of the combined datasets.

Recent developments indicate efforts to improve noise stability through integration with machine learning and deep learning-based approaches (Hu et al. 2024, 2023). These approaches have the potential to capture non-linear relationships between parameters and accelerate the inversion process. In addition to RMS, several studies have also begun to use the Structural Similarity Index Measure (SSIM) to evaluate structural similarity between the inverted and the reference model (Hu et al. 2024, 2023; Liu et al. 2024, 2023a). Unlike RMS, which is based on numerical error, SSIM assesses the similarity of structural patterns, making it more sensitive to the quality of subsurface geometric reconstruction. Nevertheless, the use of SSIM remains limited and is generally applied to synthetic data with low noise levels, indicating that these approaches are still at an early stage of development and have not yet been extensively validated under high-noise field conditions (Hu et al. 2024). This suggests that, although promising, these approaches have not yet reached methodological maturity for high-noise field applications. However, these approaches are still predominantly evaluated using synthetic data with controlled noise levels.

Overall, the results of the review indicate that CGJI has significant potential for improving stability against noise, particularly under controlled conditions. However, its performance under high-noise conditions, which more closely represent field conditions, has not yet been fully characterised. Therefore, further research is required to systematically evaluate the performance of CGJI at various noise levels, as well as to develop approaches such as advanced regularisation, adaptive weighting schemes, and the integration of data-driven methods to enhance the stability of the inversion. These efforts are crucial for expanding the application of CGJI in geophysical studies characterised by high levels of uncertainty, such as mineral and hydrocarbon exploration, as well as groundwater studies and disaster mitigation. These limitations directly inform the research gaps summarised in Table 5.

FUTURE WORK

As shown in Tables 4 and 5, the practical utility of CGJI is currently constrained by its limited validation under high-noise conditions and the lack of systematic noise evaluation frameworks. As summarised in Table 5, these gaps are not independent but reflect a common limitation in current CGJI studies, particularly the lack of systematic evaluation under realistic noise conditions. The latest trends (Zhang et al. 2019b; Zhu et al. 2023) emphasise the need for more rigorous evaluation of CGJI noise stability, particularly at noise levels above 20%. While CGJI has been shown to improve subsurface imaging by integrating multiple geophysical datasets, the true subsurface conditions remain unknown. It is therefore essential to validate that CGJI continues to produce results close to reality under high-noise scenarios.

From a theoretical perspective, the breakdown of CGJI stability at noise levels approaching 20% can be attributed to the behaviour of spatial gradient operators in the presence of stochastic fluctuations. Mathematically, the cross-gradient constraint relies on the cross-product of model gradients. As noise levels increase, the gradient of the noise begins to dominate the gradient of the actual geological signal, particularly in high-frequency components (Varfinezhad, Oskooi & Fedi 2020).

When the signal-to-noise ratio (SNR) in the gradient domain falls below a critical threshold, identified in this review as approximately 20%, the cross-gradient operator effectively couples noise artefacts rather than structural boundaries. This leads to ‘structural aliasing’, where the inversion forces the two models to share erroneous noise patterns, creating spurious correlations that do not exist in the subsurface. This phenomenon explains why many reviewed studies report a significant decline in model accuracy as they approach this threshold (Zhang et al. 2019b; Zhu et al. 2023) and underscores the limitation of using a constant weighting factor that does not account for local noise variance.

Understanding noise stability in CGJI is critical for improving inversion reliability and guiding methodological development. First, as noise directly affects inversion outcomes, algorithmic improvements, such as optimising kernel matrices, objective functions, regularisation schemes, and filters, are essential to minimise its impact without losing key information. Second, such research

TABLE 4. Summary of noise stability of CGJI based on the documents

No	Noise range	Parameters	References
1	0.1 ms random noise for the P-wave traveltimes; 1.0 ms random noise for the S-wave traveltimes	P- & S-wave velocity	(von Ketelhodt et al. 2019)
2	Gaussian noise with standard deviation of 0.02 mGal and 2 nT	Gravity (density) & Magnetic (susceptibility)	(Fregoso, Palafox & Moreles 2020)
3	1% random and Gaussian noise	P-wave velocity, Magnetotelluric (resistivity) & Resistivity	(Feng et al. 2018) (Hu et al. 2023)
4	2% Gaussian noise	Resistivity, P-wave velocity, Magnetotelluric (resistivity), Gravity (density), S-wave velocity, Magnetic (susceptibility) & Gravity gradient (density)	(Shi et al. 2017) (Saleh et al. 2022) (Zhong et al. 2022) (Huang et al. 2023) (Niu et al. 2023) (Liu et al. 2023a) (Ghari et al. 2024)
5	3% Gaussian noise	Resistivity, GPR (resistivity); Gravity (density), Magnetotelluric (resistivity), P-wave velocity & Magnetic (susceptibility)	(Jordi et al. 2020) (Liao et al. 2022) (Ma et al. 2022) (Hu et al. 2023) (de Peppo, Cercato & De Donno 2024) (Hu et al. 2024)
6	5% Gaussian noise	Control-Source Audio Magnetotelluric (resistivity), Magnetic (susceptibility), Gravity (density), Magnetotelluric (resistivity) & Induced Polarization (Chargeability)	(Wang, Tan & Wang 2017) (Zhang & Li 2019) (Zhang et al. 2019a) (Zhang et al. 2019b) (Tavakoli et al. 2021) (Li, Li & Zhang 2022) (He, Li & Zhang 2022) (Wang et al. 2022) (Zhu et al. 2023) (Hu et al. 2024)
7	10% Gaussian noise	Magnetotelluric (resistivity), Gravity (density), Gravity (density), Magnetic (susceptibility) & Gravity gradient (density)	(Zhang et al. 2019b) (Sun & Wang 2020) (Wang et al. 2022) (Hu et al. 2024)
8	15% Gaussian noise	Gravity (density) & Gravity gradient (density)	(Wang et al. 2022)
9	20% Gaussian noise	Magnetotelluric (resistivity), Gravity (density), Resistivity & Induced Polarization (Chargeability)	(Zhang et al. 2019b) (Zhu et al. 2023)

informs software developers in creating advanced tools for automated processing and stability assessment, thereby improving efficiency and subsurface image quality.

These analyses collectively indicate that while CGJI systematically improves subsurface imaging relative to standalone inversions, critical gaps remain, particularly regarding performance under high-noise conditions, comparative stability across geophysical parameters, and integration with data-driven methods such as deep learning. However, the integration of Deep Learning (DL) faces an ‘interpretability challenge,’ as these models often act as black boxes compared to deterministic physics-based constraints. Future research should therefore focus on Physics-Informed Neural Networks (PINNs) that can honor the cross-gradient mathematical framework while leveraging the robust denoising capabilities of artificial

TABLE 5. Future work of noise stability

Priority	Research gap	Future work	References
1	Noise stability more than 20%	Synthetic data test	Wang, Tan & Wang (2017) (Shi et al. 2017) (Zhang et al. 2019b) (Zhu et al. 2023) (Hu et al. 2023) (Hu et al. 2024)
2	Statistical validation	RMS, SSIM, standard deviation used in research	(Shi et al. 2017) (Hu et al. 2023) (Hu et al. 2024)
3	Real field application (Zhang et al. 2019b)	Conduct research in complex geological condition	(Hu et al. 2023) (Hu et al. 2024)

intelligent (AI). Addressing these gaps will require more rigorous statistical validation, the development of adaptive regularisation and weighting strategies to mitigate noise propagation without oversmoothing geological features, and systematic evaluation on both synthetic and real field datasets. Expanding noise assessments beyond current thresholds and extending the use of advanced algorithmic and computational strategies will strengthen CGJI's reliability and practical applicability across resource exploration, environmental monitoring, and geohazard studies.

CONCLUSION

Cross-Gradient Joint Inversion (CGJI) has established itself as a robust approach for integrating multiple geophysical datasets, offering enhanced resolution and more geologically consistent models compared to standalone inversions. Its application spans geodynamics-tectonics, hydrocarbon and mineral exploration, groundwater studies, and near-surface investigations, with gravity and magnetic data remaining the most frequently employed parameters. Recent methodological developments, including 3D extensions, stochastic approaches, advanced regularisation, and integration with deep learning, have further strengthened CGJI's versatility and computational efficiency.

Despite these advances, noise stability persists as a critical limitation. Most studies have evaluated performance under relatively low to moderate noise levels (≤ 20%), leaving uncertainty regarding stability in high-noise environments. Metrics such as RMS perturbation and SSIM indicate improved stability on CGJI's over individual inversions, yet comprehensive evaluations under realistic field conditions and higher noise thresholds are lacking. Addressing these gaps is essential for ensuring the reliability of subsurface models and for guiding practical applications in resource exploration, environmental monitoring, and disaster mitigation.

Future research should prioritise systematic assessments of CGJI under elevated noise conditions (> 20% noise), statistical validation such as SSIM, and the incorporation of data-driven frameworks to enhance both computational efficiency and model fidelity in complex geological condition. By focusing on these challenges, CGJI can realise its full potential as a reliable and versatile tool for geophysical imaging in increasingly complex and uncertain subsurface environments. Ultimately, this systematic review identifies a critical 'stability gap' and provides a foundational benchmark for researchers to transition from synthetic success to reliable field applications, ensuring more robust subsurface imaging in the modern era of multi-physics geophysics.

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